1. Consider the vectors: $\mathbf{a} = \langle 1,0,-1\rangle$ $\mathbf{b} = \langle 1,1,1\rangle$ $\mathbf{c} = \langle -1,1,0\rangle$.

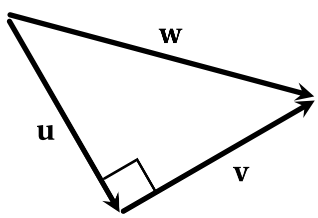
(a) Compute $\mathbf{a}\times\mathbf{b}$. **(3 points)**

$\mathbf{a}\times\mathbf{b} = \Big\langle \quad , \quad , \quad \Big\rangle$

(b) Compute the volume of the parallelepiped determined by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$. **(2 points)**

Volume =

2. Given that $\mathbf{u}$ and $\mathbf{v}$ in the picture at left have length 1, compute $\mathbf{u}\cdot\mathbf{v}$, $\mathbf{u}\cdot\mathbf{w}$, and $\text{proj}_{\mathbf{v}}\mathbf{w}$. **(1 point each)**



$\mathbf{u}\cdot\mathbf{v}$ =

$\mathbf{u}\cdot\mathbf{w}$ =

$\text{proj}_{\mathbf{v}}\mathbf{w}$ =

3. A particle moves with constant velocity $\langle 3,1,-1\rangle$ starting from the point $(3,2,4)$ at time $t = 0$. When and where will it cross the xy-plane? **(3 points)**

When: t =

Where: $\Big(\quad , \quad , \quad \Big)$

4. Let A be the plane given by $x - z = 1$ and B the plane given by $x + y + z = 2$.

(a) Find a normal vector **n** for the plane A. **(1 points)**

$\mathbf{n} = \langle \quad , \quad , \quad \rangle$

(b) Find the angle between the two planes. **(2 points)**

$\theta =$

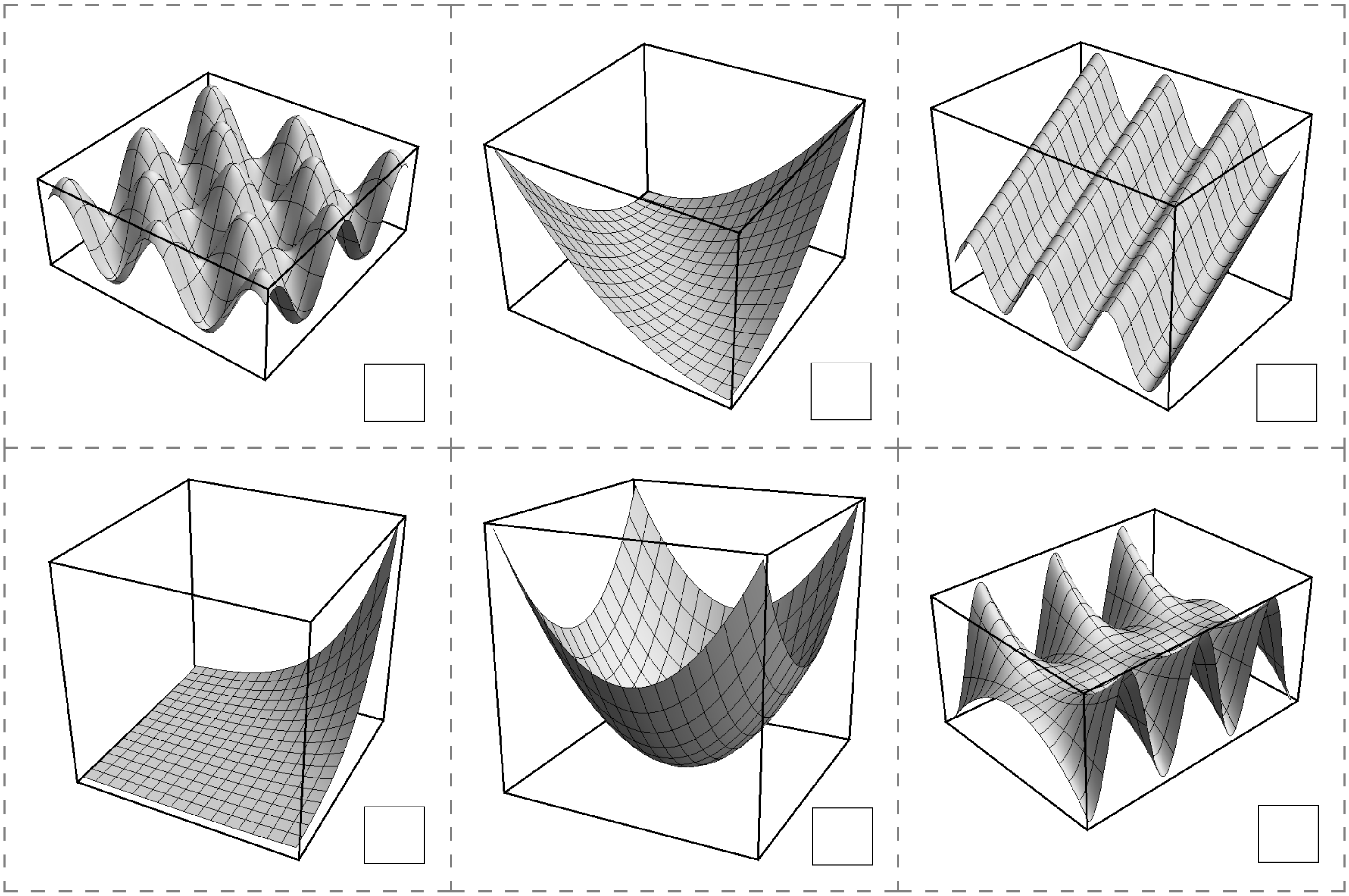
(c) Find the equation of a plane C which is perpendicular to both A and B. **(3 points)**

Equation: $\square\, x + \square\, y + \square\, z = \square$

5. Exactly one of the following two limits exists. Circle the one that exists and justify your answer. **(5 points)**

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2 - y^2}{\sqrt{x^2 + y^2}} \right) \qquad \lim_{(x,y)\to(0,0)} \left(\frac{xy}{(x^2 + y^2)^2} \right)$$

6. For each function label its graph from among the options below: (a) $(x+y)^2$ (b) $x+\cos(y)$
(3 points each)



7. Circle the equation for the quadratic surface shown at right. **(3 points)**

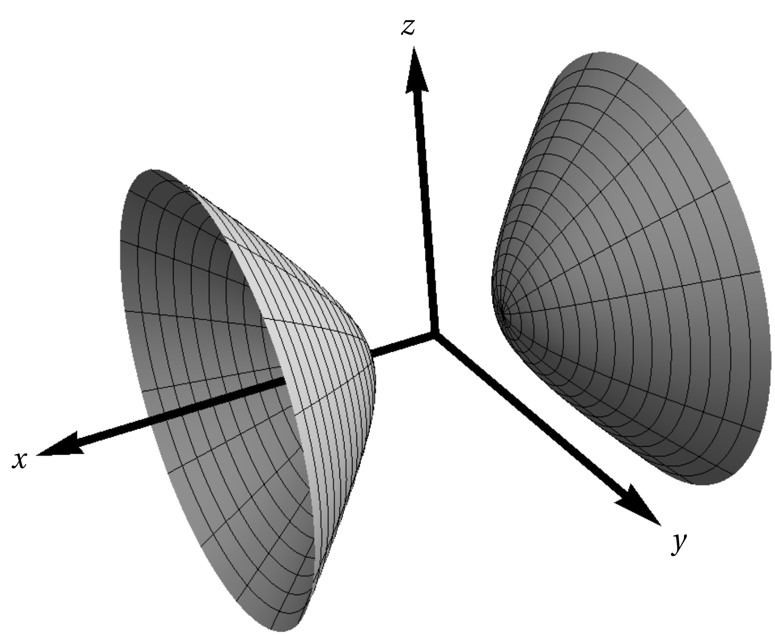
(a) $x^2+y^2+z^2=1$

(b) $x^2-y^2-z^2=-1$

(c) $x^2+y^2-z^2=-1$

(d) $x^2-y^2-z^2=1$

(e) $x-y^2-z^2=1$



8. Is the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given at right continuous at $(0,0)$? Justify your answer. **(2 points)**

$$f(x,y) = \begin{cases} x^2 + y + 1 & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

9. Let $f(x,y)$ be a function with values and derivatives in the table. Use linear approximation to estimate $f(2.1, 0.9)$. **(3 points)**

(x,y)	$f(x,y)$	$\frac{\partial f}{\partial x}(x,y)$	$\frac{\partial f}{\partial y}(x,y)$
$(-1,3)$	0	4	4
$(2,1)$	2	-1	3
$(2,4)$	3	7	7
$(3,6)$	1	-3	-5

$f(2.1, 0.9) \approx$

10. Suppose $f(x,y)$ has the contour plot below right, with points labelled. Circle the best answer to each of the following questions: **(1 point each)**

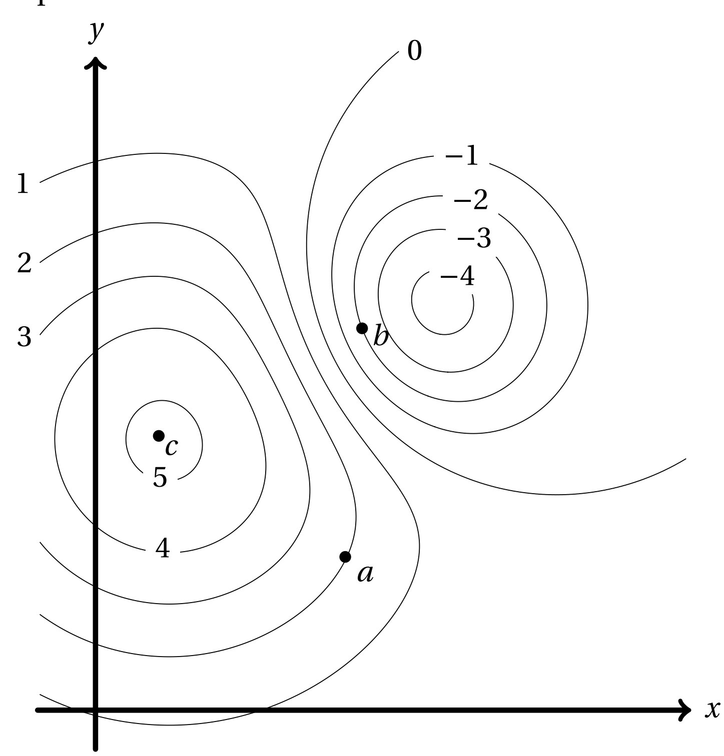
(a) $f(a)$ is: **positive** **negative** **0**

(b) $\frac{\partial f}{\partial x}(b)$ is: **positive** **negative** **0**

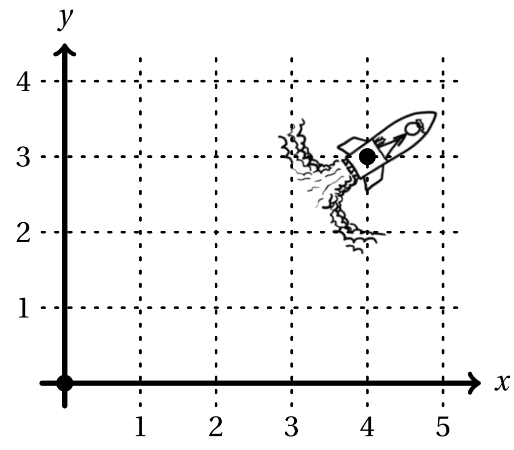
(c) $\frac{\partial^2 f}{\partial^2 y}(c)$ is: **positive** **negative** **0**

(d) Circle one:

$\frac{\partial f}{\partial x}(a) > \frac{\partial f}{\partial x}(b)$ $\qquad$ $\frac{\partial f}{\partial x}(a) < \frac{\partial f}{\partial x}(b)$



11. An exceptionally tiny spaceship positioned as shown is travelling so that its x-coordinate increases at a rate of $1/2$ m/s and y-coordinate increases at a rate of $1/3$ m/s. Use the Chain Rule to calculate the rate at which the distance between the spaceship and the point $(0,0)$ is increasing. **(6 points)**



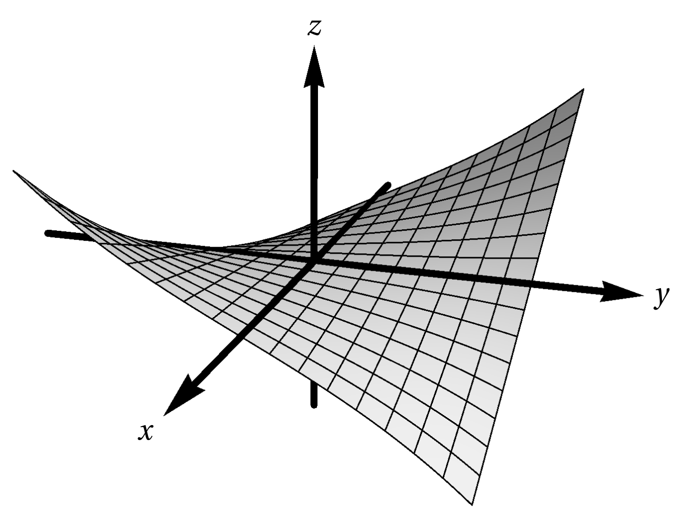
Distances in meters

Rocket courtesy of xkcd.com

rate =	m/s

12. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function whose graph is shown at right.

(a) Find the equation of the tangent plane to the graph at $(0,0,0)$. **(2 points)**



(b) The partial derivative $\dfrac{\partial^2 f}{\partial x \partial y}(0,0)$ is (circle your answer):

positive **negative** **0** **(1 point)**

13. Extra Credit Problem. Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ is continuous at $(0,0)$ with $f(0,0) = 2$ and partial derivatives $f_x(0,0) = 1$ and $f_y(0,0) = -1$. If in addition

$$\lim_{t\to 0} \frac{f(t,t)-2}{t} = 1$$

can f be differentiable at $(0,0)$? Carefully justify your answer. **(3 points)**

Scratch work may go below and on the back of this sheet.