

1. Let $A = (0, -1, 1)$, $B = (1, -1, 3)$, $C = (2, 0, 0)$ be three points.

(a) Find $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{BC}$. (2 points)

$$\mathbf{v} = \left[\begin{array}{ccc} \langle & & \rangle \\ & , & \\ & & \end{array} \right]$$

$$\mathbf{w} = \left[\begin{array}{ccc} \langle & & \rangle \\ & , & \\ & & \end{array} \right]$$

(b) Calculate the cross-product $\mathbf{v} \times \mathbf{w}$. (3 points)

$$\mathbf{v} \times \mathbf{w} = \left[\begin{array}{ccc} \langle & & \rangle \\ & , & \\ & & \end{array} \right]$$

(c) Find the area of the triangle $\triangle ABC$. (2 points)

Area(\triangle) =

2. Suppose that two planes have (non-zero) normal vectors \mathbf{n}_1 and \mathbf{n}_2 respectively, and that $\mathbf{n}_1 \times \mathbf{n}_2 = \mathbf{0}$. Which of the following could *possibly* be true? List the letters in the box. (4 points)

- A. The planes intersect in a line.
- B. The planes are orthogonal to each other.
- C. The planes are parallel to each other.
- D. The planes are equal to each other.

The possibly true statements are

3. (a) Let $\mathbf{v} = \langle 1, 0, 2 \rangle$, and let $\mathbf{w} = \langle -1, 3, 0 \rangle$. Find $\text{proj}_{\mathbf{v}} \mathbf{w}$, the vector projection of \mathbf{w} onto \mathbf{v} . (3 points)

$$\text{proj}_{\mathbf{v}} \mathbf{w} = \left\langle \quad, \quad, \quad \right\rangle$$

(b) Let \mathbf{P} be the plane with equation $x + 2z = 0$. Find the distance from the point $(-1, 3, 0)$ to the plane \mathbf{P} . (2 points)

The distance is

4. Find the angle $0 \leq \theta \leq \frac{\pi}{2}$ between the planes $z = x + \sqrt{2}y$ and $x - z = 5$.
Express your answer in radians. **(4 points)**

$\theta =$

5. Find the tangent plane to the surface $z = x^2 e^y$ at the point $(3, 0, 9)$. **(5 points)**

Equation: $z =$

$x +$

$y +$

6. Suppose f is a differentiable function of x and y with continuous second partial derivatives. Let $g(u, v) = f(e^u + (v+2)^2, e^{3u} + v^3)$. You are given the following table of values.

	g	f	f_x	f_y	f_{xx}	f_{xy}
$(0, 0)$	5	1	4	6	4	5
$(5, 1)$	0	4	7	2	1	9

- (a) Use the table to calculate $g_u(0, 0)$, if possible.
Otherwise, write "Insufficient information". **(4 points)**

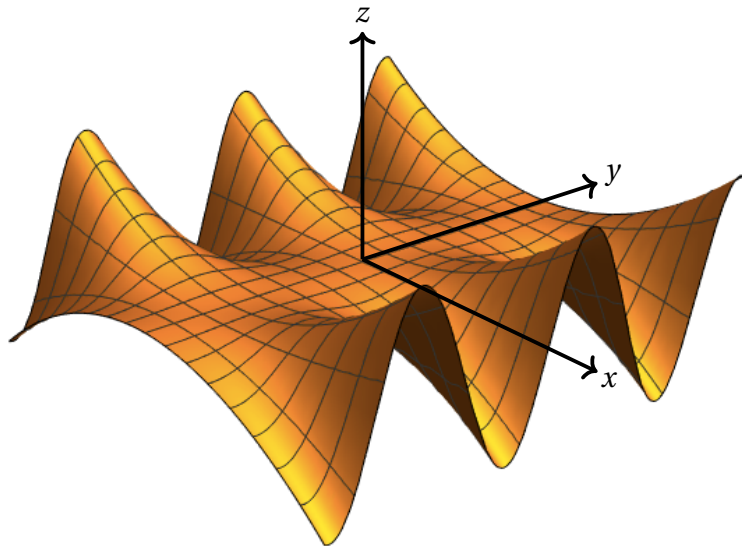
$$g_u(0, 0) =$$

- (b) Use the table to calculate $f_{yx}(5, 1)$, if possible.
Otherwise, write "Insufficient information". **(1 point)**

$$f_{yx}(5, 1) =$$

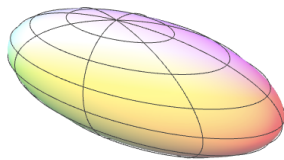
7. The picture below is the graph of a function $z = f(x, y)$ illustrated relative to the coordinate axes. Pick the correct function f . (2 points)

- A. $f(x, y) = x^2 + y^2 - 2$
- B. $f(x, y) = x^2 \cos(y)$
- C. $f(x, y) = x^2 \sin(y)$
- D. $f(x, y) = xy e^{xy}$
- E. $f(x, y) = \sin(x) \cos(y)$

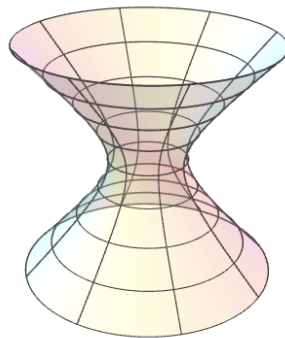


The correct function is

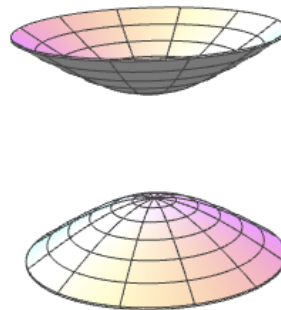
8. Let $f(x, y, z) = ax^2 + by^2 + cz^2$ for some real numbers a , b , and c . Which of the following *could not* be a level set of f ? Circle the letter corresponding to your answer. (2 points)



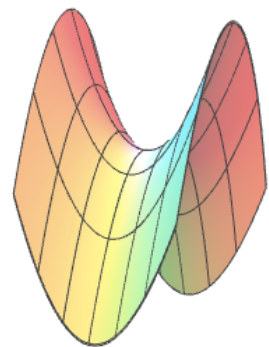
A. Ellipsoid



B. Hyperboloid of 1 sheet



C. Hyperboloid of 2 sheets



D. Hyperbolic paraboloid

9. Let f be a function of x and y . Consider the following statements.

- A. $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ along every straight line through $(0, 0)$, but $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ does not exist.
- B. $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ along the lines $x = 0$ and $y = 0$, but $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ does not exist.
- C. $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ along the lines $x = 0$ and $y = 0$, and $\lim_{(x, y) \rightarrow (0, 0)} f(x, y) = 1$.

Which statements could *possibly* be true?

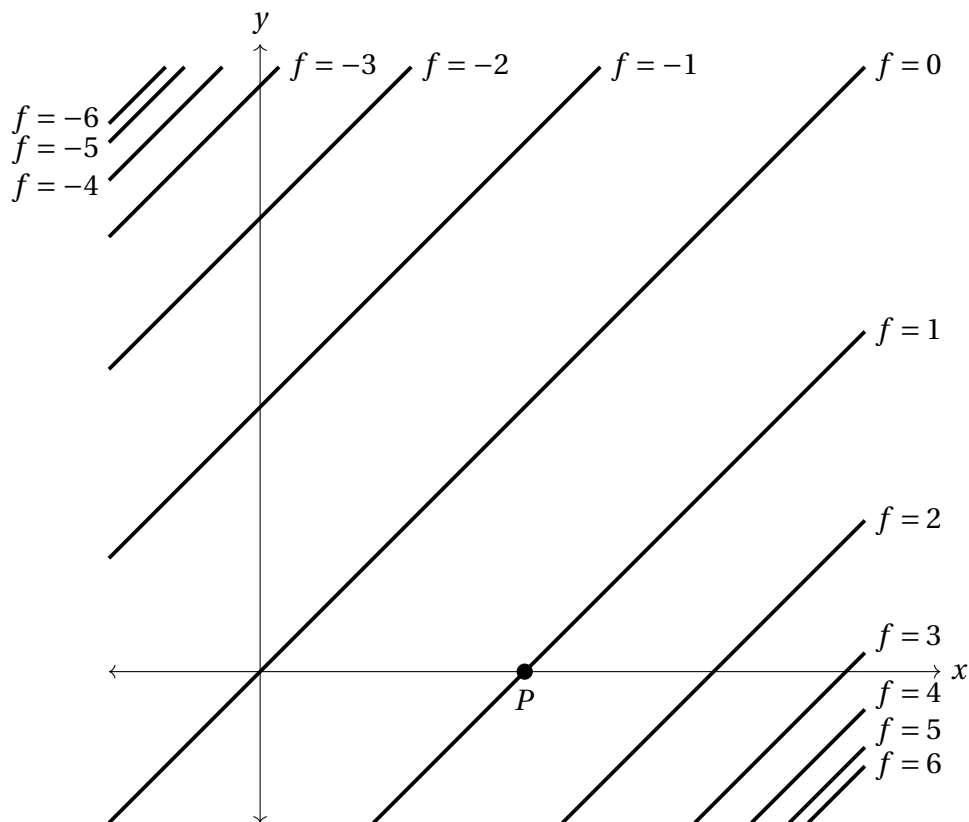
List the letter(s) for those statement(s) in the box, or write "none". (3 points)

The possibly true statements are

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10. A contour map for a function f of x and y and a point P in the plane are given below.

Use the contour map to determine if the following quantities are negative, zero, or positive. (2 points each)



(a) $f_x(P)$ is negative zero positive

(b) $f_{xx}(P)$ is negative zero positive

(c) $f_{xy}(P)$ is negative zero positive.