1. Let $A = (0,-1,1), B = (1,-1,3), C = (2,0,0)$ be three points.

(a) Find $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{BC}$. **(2 points)**

$v = \langle 1,-1,3\rangle - \langle 0,-1,1\rangle = \langle 1,0,2\rangle$
$w = \langle 2,0,0\rangle - \langle 1,-1,3\rangle = \langle 1,1,-3\rangle.$

$\mathbf{v} = \boxed{\langle 1, 0, 2\rangle}$ $\mathbf{w} = \boxed{\langle 1, 1, -3\rangle}$

(b) Calculate the cross-product $\mathbf{v}\times\mathbf{w}$. **(3 points)**

The cross-product is given by

$$v\times w = \begin{vmatrix} i & j & k \\ 1 & 0 & 2 \\ 1 & 1 & -3 \end{vmatrix} = \begin{vmatrix} 0 & 2 \\ 1 & -3 \end{vmatrix} i - \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} j + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} k =$$
$$(0(-3) - 1(2))i - (1(-3) - 1(2))j + (1(1) - 1(0))k$$
$$= -2i + 5j + k$$

$\mathbf{v}\times\mathbf{w} = \boxed{\langle -2, 5, 1\rangle}$

(c) Find the area of the triangle $\triangle ABC$. **(2 points)**

The magnitude of the cross product gives the area of the parallelogram determined by the vectors v and w.
The magnitude of $v\times w$ is given by $\sqrt{(-2)^2+(5)^2+1^2} = \sqrt{4+25+1} = \sqrt{30}$.
The area of the triangle is half of this, which is $\frac{\sqrt{30}}{2}$.

Area(Δ) = $\boxed{\frac{\sqrt{30}}{2}.}$

2. Suppose that two planes have (non-zero) normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ respectively, and that $\mathbf{n}_1 \times \mathbf{n}_2 = \mathbf{0}$. Which of the following could *possibly* be true? List the letters in the box. **(4 points)**

A. The planes intersect in a line.

B. The planes are orthogonal to each other.

C. The planes are parallel to each other.

D. The planes are equal to each other.

Having cross product 0 means the normal vectors for the planes are parallel. Having parallel normal vectors corresponds to having parallel planes or the same plane, so only C and D are possible.

The possibly true statements are $\boxed{\text{C and D}}$

3. (a) Let $\mathbf{v} = \langle 1,0,2\rangle$, and let $\mathbf{w} = \langle -1,3,0\rangle$. Find $\text{proj}_{\mathbf{v}}\mathbf{w}$, the vector projection of $\mathbf{w}$ onto $\mathbf{v}$. **(3 points)**

We have that $v \cdot v = |v|^2 = \left(\sqrt{1^2+2^2}\right)^2 = 5$ and $v \cdot w = 1(-1)+0+0 = -1$.

Thus, $\text{proj}_v w = \left(\frac{v \cdot w}{v \cdot v}\right) v = -\frac{1}{5}\langle 1,0,2\rangle = \langle -\frac{1}{5}, 0, \frac{-2}{5}\rangle$

$\text{proj}_{\mathbf{v}}\mathbf{w} = \boxed{\left\langle -\frac{1}{5}, 0, -\frac{2}{5} \right\rangle}$

(b) Let $\mathbf{P}$ be the plane with equation $x + 2z = 0$. Find the distance from the point $(-1,3,0)$ to the plane $\mathbf{P}$. **(2 points)**

Note that v from above is a normal vector to this plane. Additionally, the plane passes through the origin $(0,0,0)$. Finally, the vector w gives the vector from $(0,0,0)$ to $(-1,3,0)$. So, $|\text{proj}_v w|$ will give the distance from the point to the plane.

It is $|\text{proj}_v w| = \sqrt{\frac{1}{25}+\frac{4}{25}} = \frac{1}{\sqrt{5}}$

The distance is $\boxed{1/\sqrt{5}}$

4. Find the angle $0 \le \theta \le \frac{\pi}{2}$ between the planes $z = x + \sqrt{2}y$ and $x - z = 5$.
Express your answer in radians. **(4 points)**

This is equivalent to finding the angle between normal vectors to the planes. Two such vectors are $\langle 1, \sqrt{2}, -1\rangle$ for the first one and $\langle 1, 0, -1\rangle$ for the second one. Recall that $\cos\theta = \dfrac{\langle 1,\sqrt{2},-1\rangle \cdot \langle 1,0,-1\rangle}{|\langle 1,\sqrt{2},-1\rangle||\langle 1,0,-1\rangle|} = \frac{1}{\sqrt{2}}$. Thus, $\theta = \pi/4$.

$\theta =$ $\boxed{\pi/4.}$

5. Find the tangent plane to the surface $z = x^2 e^y$ at the point $(3, 0, 9)$. **(5 points)**

Computing the following partial derivatives will complete this task:

$f_x = 2xe^y$

$f_y = x^2 e^y$

$f_x(3,0) = 6$

$f_y(3,0) = 9.$

Thus, the equation is given by

$z = 9 + 6(x-3) + 9(y-0) = 6x + 9y - 9$

Equation: $z = \boxed{6}\, x + \boxed{9}\, y + \boxed{-9}$

6. Suppose f is a differentiable function of x and y with continuous second partial derivatives. Let $g(u,v) = f(e^u + (v+2)^2, e^{3u} + v^3)$. You are given the following table of values.

	g	f	f_x	f_y	f_{xx}	f_{xy}
(0,0)	5	1	4	6	4	5
(5,1)	0	4	7	2	1	9

(a) Use the table to calculate $g_u(0,0)$, if possible.
Otherwise, write "Insufficient information". **(4 points)**

Let $x = e^u + (v+2)^2$, $y = e^{3u} + v^3$.

Then, compute:

$x_u = e^u$, $y_u = 3e^{3u}$, $x(0,0) = 5$, $y(0,0) = 1$, $x_u(0,0) = 1$, and $y_u(0,0) = 3$.

Thus, $g_u(0,0) = f_x(5,1)x_u(0,0) + f_y(5,1)y_u(0,0) = (7)(1) + (2)(3) = 13$

$g_u(0,0) =$ 13

(b) Use the table to calculate $f_{yx}(5,1)$, if possible.
Otherwise, write "Insufficient information". **(1 point)**

This will equal $f_{xy}(5,1)$ by Clairaut's theorem, so it is 9.

$f_{yx}(5,1) =$ 9

7. The picture below is the graph of a function $z = f(x, y)$ illustrated relative to the coordinate axes. Pick the correct function f. **(2 points)**

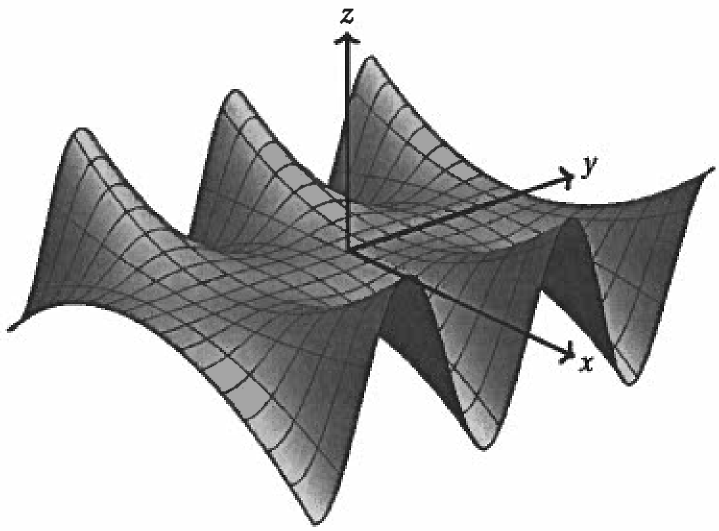
A. $f(x, y) = x^2 + y^2 - 2$

B. $f(x, y) = x^2 \cos(y)$

C. $f(x, y) = x^2 \sin(y)$

D. $f(x, y) = xye^{xy}$

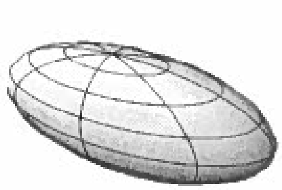
E. $f(x, y) = \sin(x)\cos(y)$



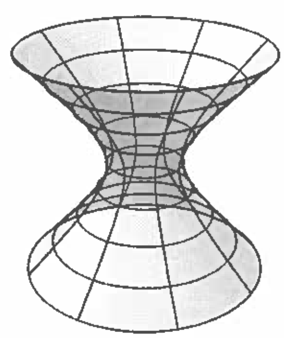
The way to solve this is to test points to see if they are on the graph. For instance, the function for A does not work because the point (0,0,-2) is not on the graph. Thus, the answer is C by process of elimination

The correct function is $\boxed{C}$

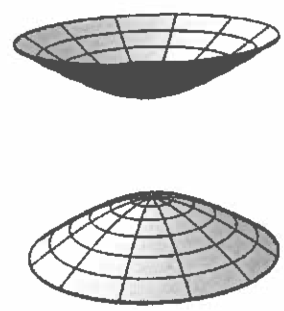
8. Let $f(x, y, z) = ax^2 + by^2 + cz^2$ for some real numbers a, b, and c. Which of the following *could not* be a level set of f? Circle the letter corresponding to yours answer. **(2 points)**



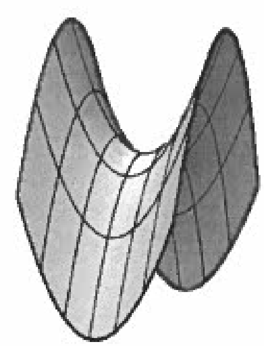
A. Ellipsoid



B. Hyperboloid of 1 sheet



C. Hyperboloid of 2 sheets



D. Hyperbolic paraboloid

The way to solve this is to look at the equations for each of these. The only one for which ~~the~~ the equation does not match up is the hyperbolic paraboloid. D

9. Let f be a function of x and y. Consider the following statements.

A. $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along every straight line through $(0,0)$, but $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

B. $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the lines $x=0$ and $y=0$, but $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

C. $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the lines $x=0$ and $y=0$, and $\lim_{(x,y)\to(0,0)} f(x,y) = 1$.

Which statements could *possibly* be true?
List the letter(s) for those statement(s) in the box, or write "none". **(3 points)**

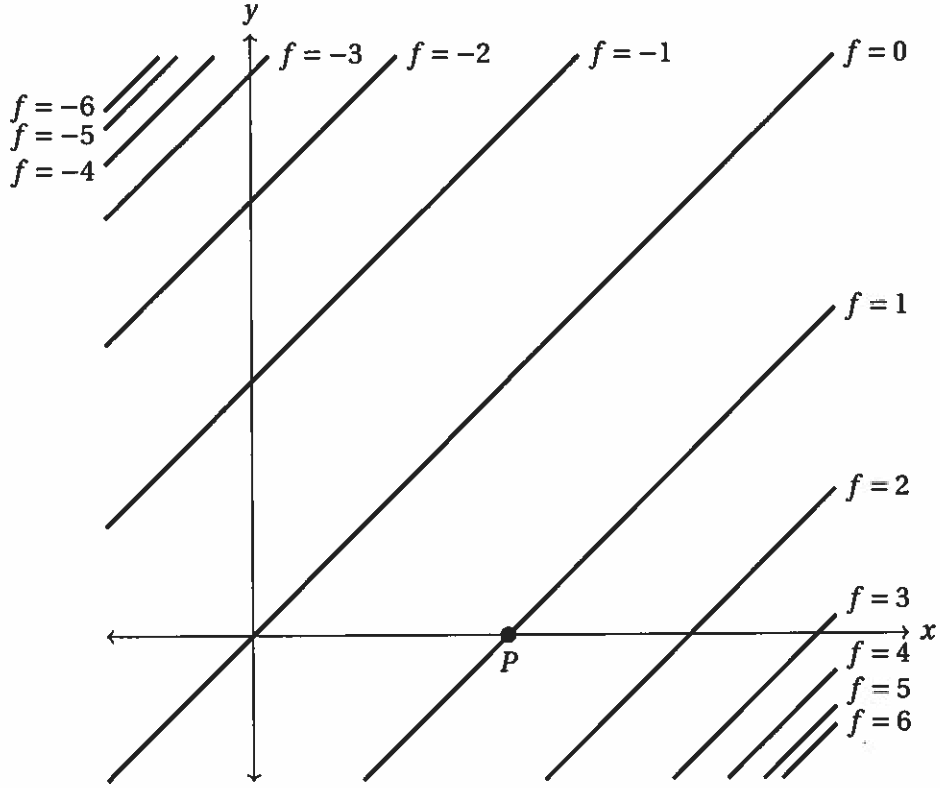
The possibly true statements are $\boxed{\text{A and B}}$

A is possible by using the function $f(x,y) = \frac{x^2y}{x^4+y^2}$. This also works for B.

C is impossible by the definition of taking limits.

10. A contour map for a function f of x and y and a point P in the plane are given below.

Use the contour map to determine if the following quantities are negative, zero, or positive. **(2 points each)**



(a) $f_x(P)$ is | negative | zero | positive |

(b) $f_{xx}(P)$ is | negative | zero | positive |

(c) $f_{xy}(P)$ is | negative | zero | positive |

Note that $\frac{\partial f}{\partial x}$ is the rate of change in the x direction. Imagine walking from P in the direction of the positive x-axis. Comparing the values of the level sets of f before and after travelling from P gives that $f_x(P) > 0$.

For $f_{xx}(P)$, note that the values of the level sets are increasing with the gaps between them decreasing, so $f_{xx}(P) > 0$.

For $f_{xy}(P)$, similar analysis shows that $f_{xy}(P) < 0$. Think of how $\frac{\partial f}{\partial x}$ changes as you move in the positive y-direction.