1. Suppose $f(x,y)$ has values and partial derivatives as in the table at right.

(a) Find all the critical points you can from the given data and classify them into local mins, local maxes, and saddles. **(3 points)**

(x,y)	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
(1,0)	2	0	1	1	0	2
(0,1)	3	0	0	−3	1	−2
(2,0)	0	−1	1	1	0	2
(1,2)	5	0	0	−1	3	2

Local mins (if any):
Local maxes (if any):
Saddles (if any):

(b) Let $L(x,y)$ denote the linear approximation to $f(x,y)$ at the point $(0,1)$. Is $L(0.1,\,1.1)$ likely to be larger than, equal to, or smaller than $f(0.1,\,1.1)$? Circle your answer below **(1 point)**

$L(0.1,\,1.1) > f(0.1,\,1.1)$	$L(0.1,\,1.1) = f(0.1,\,1.1)$	$L(0.1,\,1.1) < f(0.1,\,1.1)$

2. Find an equation of the tangent plane to the surface defined by the equation $\frac{1}{2}x^2 + xy + 3yz^2 = 7$ at the point $(2,1,1)$. **(3 points)**

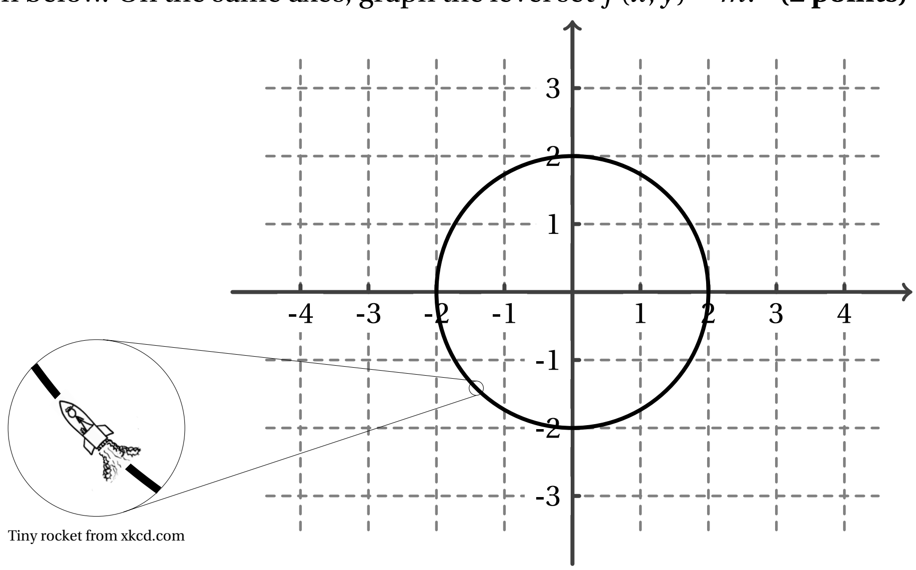
Equation: $\square\, x + \square\, y + \square\, z = \square$

3. A tiny spaceship is orbiting on the path given by $x^2 + y^2 = 4$. The solar radiation at a point (x, y) in the plane is $f(x, y) = (x+2)y = xy + 2y$.

(a) Use the method of Lagrange multipliers to find the maximum value M and minimum value m of solar radiation experienced by the tiny spaceship in its orbit. **(5 points)**

$M =$	$m =$

(b) The orbit of the spaceship is shown below. On the same axes, graph the level set $f(x, y) = m$. **(2 points)**



Tiny rocket from xkcd.com

4. Let C be the curve in $\mathbb{R}^3$ parameterized by $\mathbf{r}(t) = \langle \sin t,\ 2t,\ \cos t \rangle$ for $0 \leq t \leq \pi/2$.

(a) Compute the length of C. **(3 points)**

Length =

(b) Evaluate the integral $\int_C x^2 z \, ds$. **(4 points)**

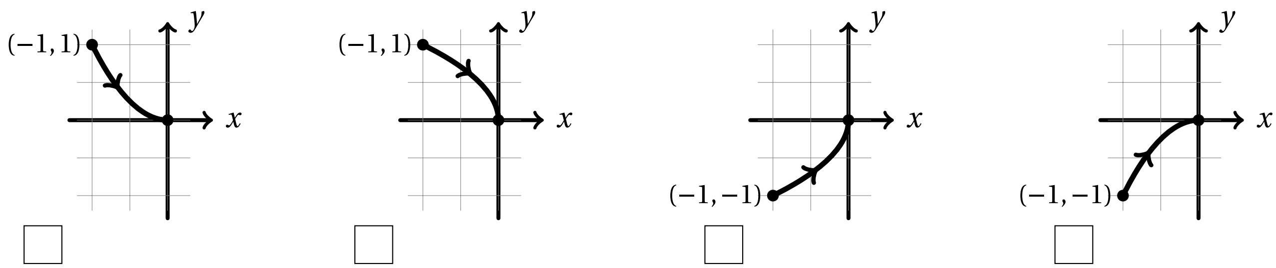
$\int_C x^2 z \, ds$ =

(c) What is the average value of $f(x, y, z) = x^2 z$ on the curve C? **(1 point)**

Average =

5. Let C be the curve in $\mathbb{R}^2$ parameterized by $\mathbf{r}(t) = \langle -t^2, t \rangle$ for $-1 \le t \le 0$.

(a) Mark the picture of C from among the choices below. **(1 point)**



(b) For the vector field $\mathbf{F} = \langle y,\ x+3 \rangle$ directly calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using the given parameterization. **(4 points)**

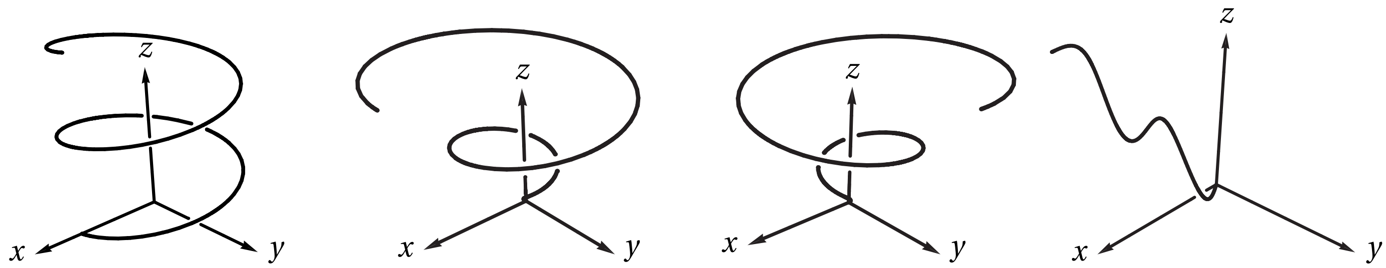
$\int_C \mathbf{F} \cdot d\mathbf{r} =$

(c) The vector field $\mathbf{F}$ is conservative. Find $f\colon \mathbb{R}^2 \to \mathbb{R}$ with $\nabla f = \mathbf{F}$. **(2 points)**

$f(x, y) =$

(d) Use your answer in part (c) to check your answer from part (b). **(2 points)**

6. (a) Mark the picture of the curve in $\mathbb{R}^3$ parameterized by $\mathbf{r}(t) = \langle t\cos t,\ t\sin t,\ t\rangle$ for $0 \le t \le 4\pi$. **(2 points)**



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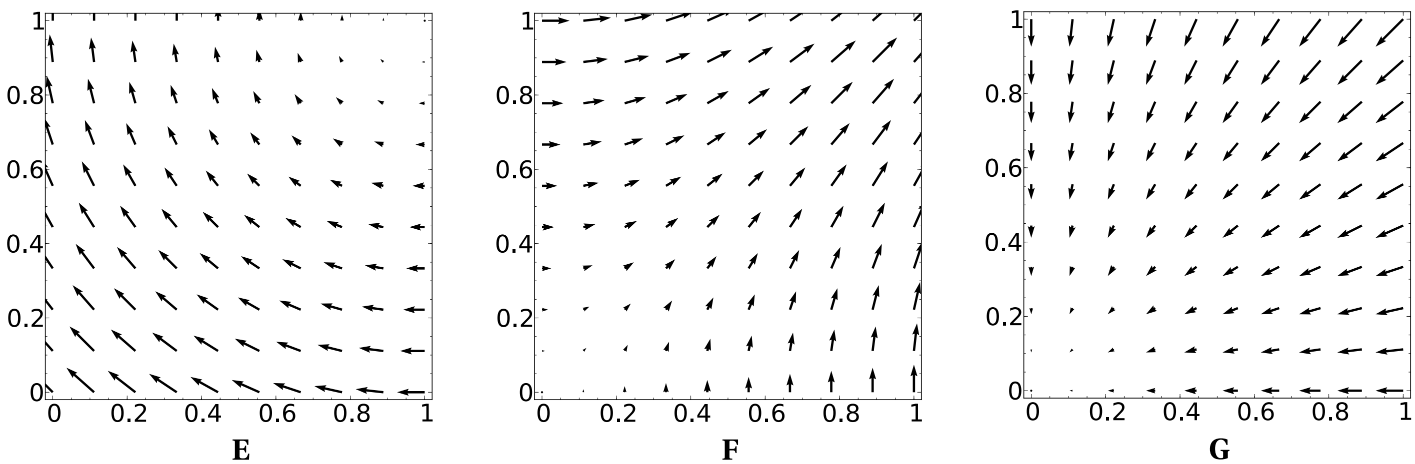
(b) Consider the three vector fields **E**, **F**, and **G** on $\mathbb{R}^2$ shown below.



(i) One of these vector fields is $y\mathbf{i} + x\mathbf{j}$. Circle its name here: | **E** **F** **G** | **(1 points)**

(ii) Exactly one of these vector fields is **not** conservative. Circle it here: | **E** **F** **G** | **(1 points)**

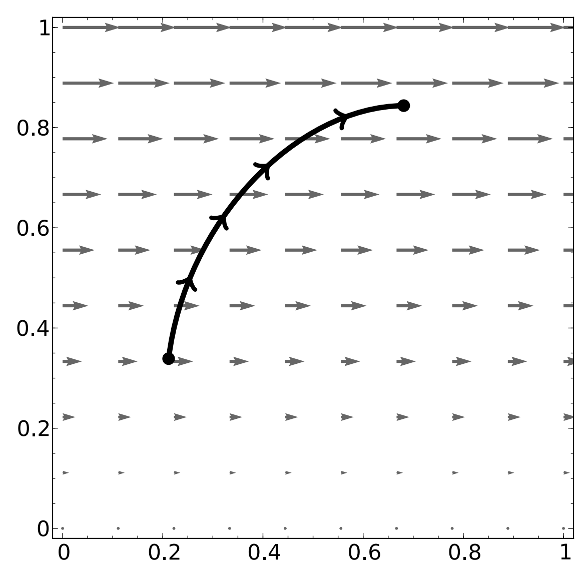
(iii) Exactly one of the following is a flowline (also called a streamline or integral curve) for **G** parameterized by time for $0 \le t \le 1$. Circle it. **(1 point)**

$\mathbf{r}(t) = \langle t, 1-t\rangle$	$\mathbf{r}(t) = \left\langle e^{-t}, e^{-t}\right\rangle$	$\mathbf{r}(t) = \left\langle t, \sqrt{t}\right\rangle$	$\mathbf{r}(t) = \langle 1-t, 1-t\rangle$

(c) Consider the curve C and vector field **H** shown at right. Is the integral $\displaystyle\int_C \mathbf{H}\cdot d\mathbf{r}$:

positive	negative	zero

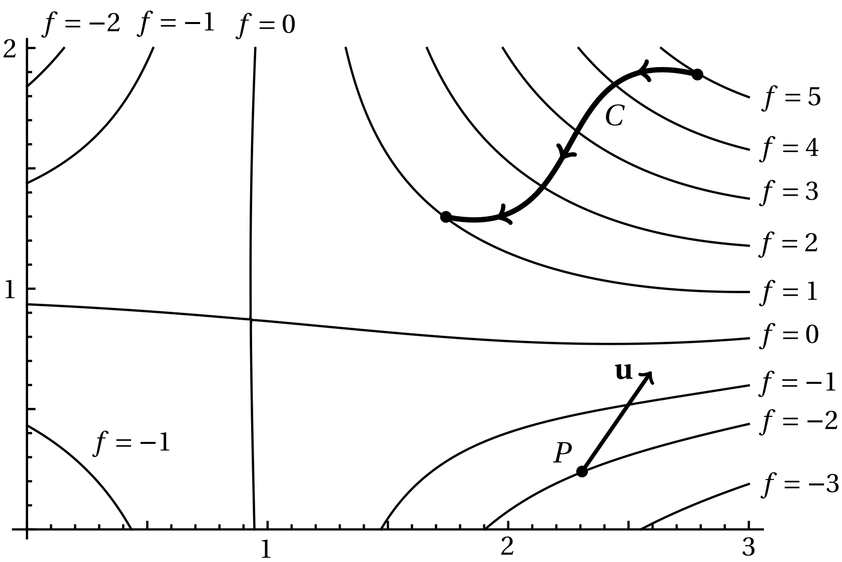
(1 points)



7. Find a parameterization $\mathbf{r}(t)$ of the curve of intersection of the cylinder $(x-1)^2 + y^2 = 1$ and the plane $x - y + z = 1$; specify the range of the parameter t. **(4 points)**

$\mathbf{r}(t) = \langle \quad , \quad , \quad \rangle$ for $\square \le t \le \square$

8. Consider the function $f(x, y)$ on the rectangle $D = \{0 \le x \le 3 \text{ and } 0 \le y \le 2\}$ whose contours are shown below right. For each part, circle the best answer. **(1 point each)**



(a) The value of $D_{\mathbf{u}} f(P)$ is:

negative   zero   positive

(b) The number of critical points of f in D is:

0   1   2   3

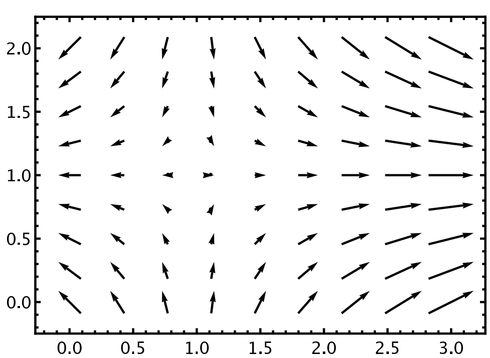
(c) The integral $\int_C f \, ds$ is:

negative   zero   positive

(d) The integral $\int_C \nabla f \cdot d\mathbf{r}$ is:

−4   −2   0   2   4

(e) Mark the plot below of the gradient vector field ∇f.



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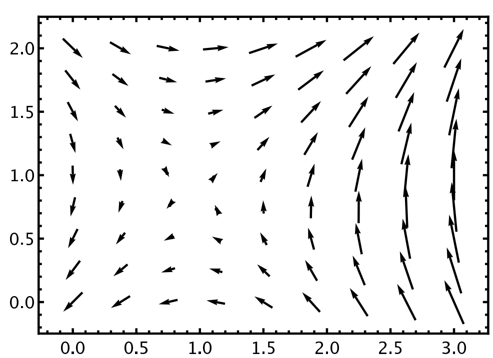

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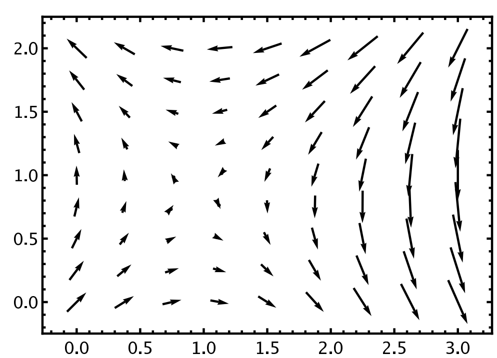

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