1. Find the maximum value of the function $f(x, y) = 3x + y$ on the curve $x^2 + y^2 = 10$. **(5 points)**

Max. value =

2. The table below contains data about a differentiable function $g(x,y)$ at several points. Find all the critical points from among the points listed below, and determine whether each is a local maximum, local minimum, or saddle point.

(x,y)	$g(x,y)$	$g_x(x,y)$	$g_y(x,y)$	$g_{xx}(x,y)$	$g_{yy}(x,y)$	$g_{xy}(x,y)$
$(-2,0)$	-16	0	0	12	2	0
$(0,0)$	0	12	0	0	2	0
$(2,0)$	16	0	0	-12	2	0

For each of these points, circle the phrase that makes the sentence true. **(1 point each)**

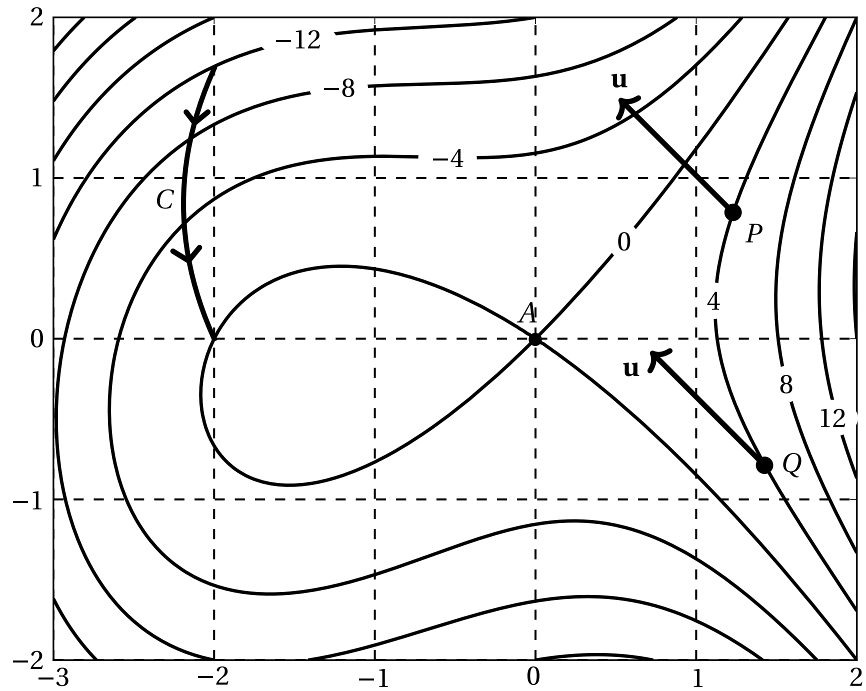
$(-2,0)$ is	not a critical point	a local minimum	a local maximum	a saddle point	of g.
$(0,0)$ is	not a critical point	a local minimum	a local maximum	a saddle point	of g.
$(2,0)$ is	not a critical point	a local minimum	a local maximum	a saddle point	of g.

3. The vector field $\mathbf{F} = \langle y^2, 2xy + 3y^2 \rangle$ is conservative. Find a function $f: \mathbb{R}^2 \to \mathbb{R}$ such that $\mathbf{F} = \nabla f$. **Note**: Check your answer—no partial credit will be given on this problem. **(3 points)**

$f(x,y) =$

Scratch space below

4. The contour plot of a differentiable function f is shown below. **(2 points each)**



(a) Circle the phrase that makes this sentence true: "The point A is __________ of f."

not a critical point	a local maximum	a local minimum	a saddle point

(b) Circle the statement which best describes the relationship between the directional derivatives $D_{\mathbf{u}}f(P)$ and $D_{\mathbf{u}}f(Q)$, where $\mathbf{u}$ is the unit vector indicated at the points P and Q.

$D_{\mathbf{u}}f(P) > D_{\mathbf{u}}f(Q)$	$D_{\mathbf{u}}f(P) < D_{\mathbf{u}}f(Q)$	$D_{\mathbf{u}}f(P) = D_{\mathbf{u}}f(Q)$

(c) For C the oriented curve shown above, evaluate the line integral $\int_C \nabla f \cdot d\mathbf{r}$.

−12	−9	−6	−3	0	3	6	9	12

(d) Estimate the value of $\int_C f(x, y)\, ds$:

−12	−9	−6	−3	0	3	6	9	12

5. Which one of the following vector fields is conservative? Circle your answer. In each case the domain is the set of points where the formula makes sense. **(2 points)**

$\left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$	$\langle x^2, y^2 \rangle$	$\langle y^2, x^2 \rangle$	$\langle xy, xy \rangle$

6. Let $f(x, y)$ be a differentiable function on the disk $D = \{x^2 + y^2 \leq 100\}$ in $\mathbb{R}^2$, where

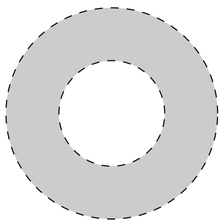
- $f(x, y) = 20$ for every point (x, y) on the circle $x^2 + y^2 = 100$.
- $f(0,0) = 5$.
- f has only one critical point, which is at $(1,2)$.

Decide which of the four statements below is true and mark the box next to it. **(2 points)**

☐ $f(1,2) > 5$ ☐ $f(1,2) < 5$ ☐ $f(1,2) = 5$

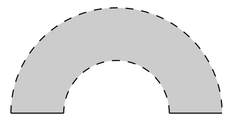
☐ The relationship between $f(1,2)$ and 5 cannot be determined from the given information.

7. Consider the following four regions in the plane: **(1 point each)**



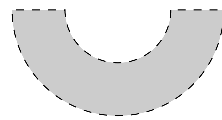
$R_1 = \{1 < x^2 + y^2 < 4\}$

$R_2 = \{y \geq 0\}$



$R_3 = \left\{\begin{array}{c} 1 < x^2 + y^2 < 4 \\ \text{and } y \geq 0 \end{array}\right\}$

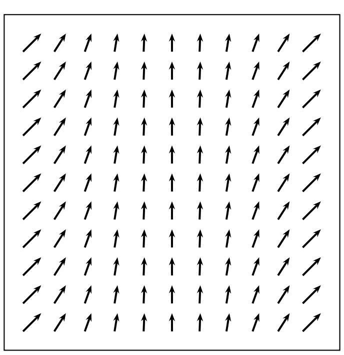
$R_4 = \left\{\begin{array}{c} 1 < x^2 + y^2 < 4 \\ \text{and } y < 0 \end{array}\right\}$



(a) Which region is neither open nor closed? R_1 R_2 R_3 R_4

(b) Which region is not simply connected? R_1 R_2 R_3 R_4

8. Mark the box below the only one of these three vector fields which is **not** conservative. **(2 points)**



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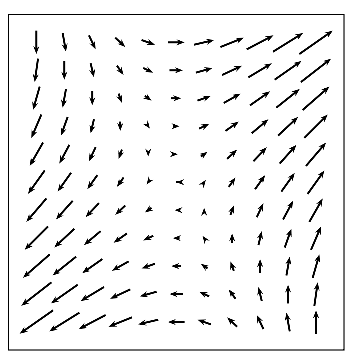

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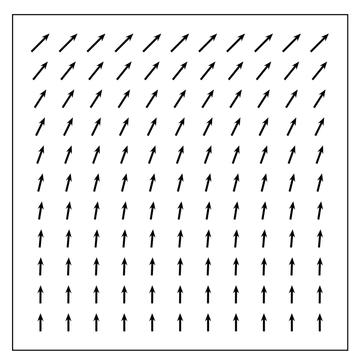

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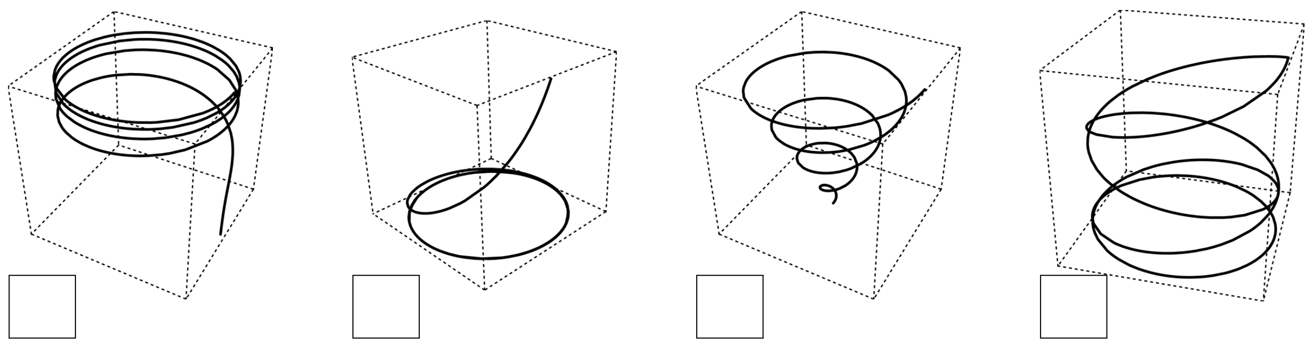
9. Find a vector function $\mathbf{r}(t)$ that represents the curve of intersection of the cylinder $y^2 + z^2 = 4$ and the hyperbolic paraboloid $x = z^2 - y^2$. Specify the range of the parameter t so that your answer traces the curve exactly once. **(4 points)**

$\mathbf{r}(t) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$ for $\square \le t \le \square$

10. Let C be the oriented curve parameterized by $\mathbf{r}(t) = \langle \cos t, \sin t, e^t \rangle, 0 \le t \le 8\pi$.

(a) Check the box next to the picture which best matches C. **(2 points)**



(b) Calculate the line integral $\int_C z\,\mathrm{d}z$. **(4 points)**

$\displaystyle\int_C z\,\mathrm{d}z =$

11. Consider the surface S defined by the equation $x^3 + y^3 + z^3 = -8$. Find an equation for the plane tangent to S at the point $(1, -1, -2)$. **(4 points)**

Equation: $\boxed{}\, x + \boxed{}\, y + \boxed{}\, z = \boxed{}$

12. Find the mass of a thin wire in the shape of the curve $x = \sin t,\ y = 2\sin t,\ z = \sqrt{5}\cos t,\ 0 \le t \le \pi$, if the wire has density function $\rho(x, y, z) = x$. **(5 points)**

Total mass =