1. Consider the vector field $\mathbf{F}(x, y) = \langle\, y + e^x,\ x - \cos y \,\rangle$. Find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$. **(2 points)**

$f(x, y) =$

2. For each of the given regions D in $\mathbb{R}^2$ below, circle the phrase that makes the sentence true. **(1 point each)**

(a) A continuous function on $D = \{x^2 + 4y^2 \geq 5\}$

must	might or might not	must not

have an absolute maximum.

(b) A continuous function on $D = \{x^2 + 4y^2 \leq 5\}$

must	might or might not	must not

have an absolute maximum.

(c) A continuous function on $D = \{x^2 + 4y^2 < 5\}$

must	might or might not	must not

have an absolute maximum.

(d) A continuous function on $D = \{x^2 + 4y^2 = 5\}$

must	might or might not	must not

have an absolute maximum.

3. Suppose $f(x, y)$ is a differentiable function with continuous second order partial derivatives and values given by the table below.

(x, y)	$f(x, y)$	$f_x(x, y)$	$f_y(x, y)$	$f_{xx}(x, y)$	$f_{yy}(x, y)$	$f_{xy}(x, y)$
$(-1, 0)$	4	0	0	-2	-3	2
$(0, 1)$	0	0	1	1	2	0
$(2, 1)$	-2	0	0	1	1	3

For each of the given points, circle the best description of the point. **(1 point each)**

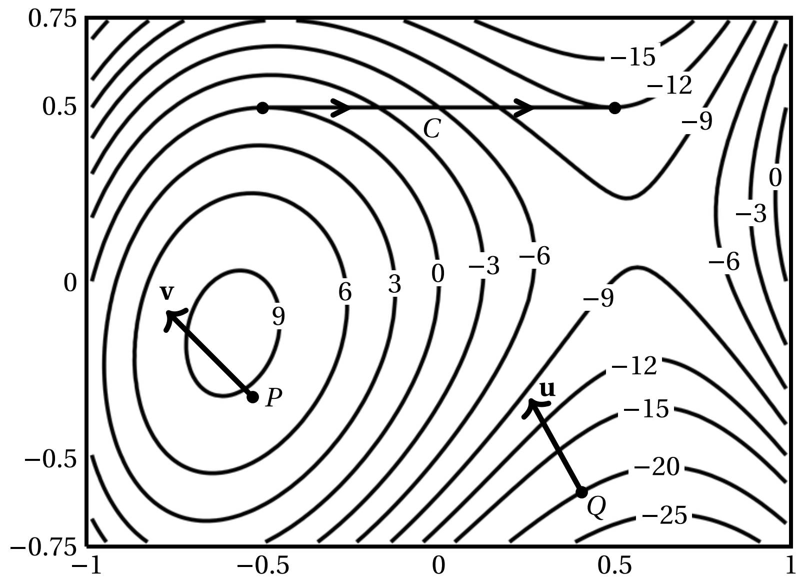
$(-1, 0)$	not critical	local minimum	local maximum	saddle point	undetermined
$(0, 1)$	not critical	local minimum	local maximum	saddle point	undetermined
$(2, 1)$	not critical	local minimum	local maximum	saddle point	undetermined

4. Find the maximum and minimum values of the function $f(x, y) = -x + 2y$ on the curve $x^2 + 2y^2 = 3$.
(5 points)

Minimum value $=$

Maximum value $=$

5. The contour map of a differentiable function g is shown at right. For each part, circle the best answer. **(2 points each)**



(a) The directional derivative $D_{\mathbf{v}}g(P)$ is:

positive	negative	zero

(b) The vector $\mathbf{u}$ is parallel to $\nabla g(Q)$.

True	False

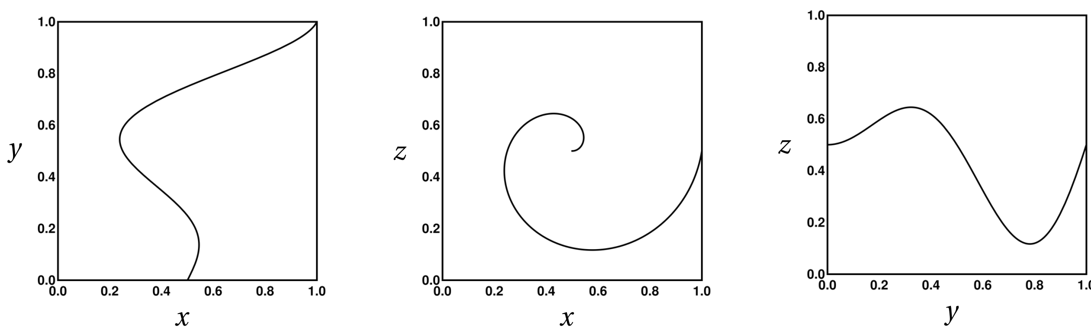
(c) Estimate $\int_C g(x, y)\, ds$.

−12	−9	−6	−3	0	3	6	9	12

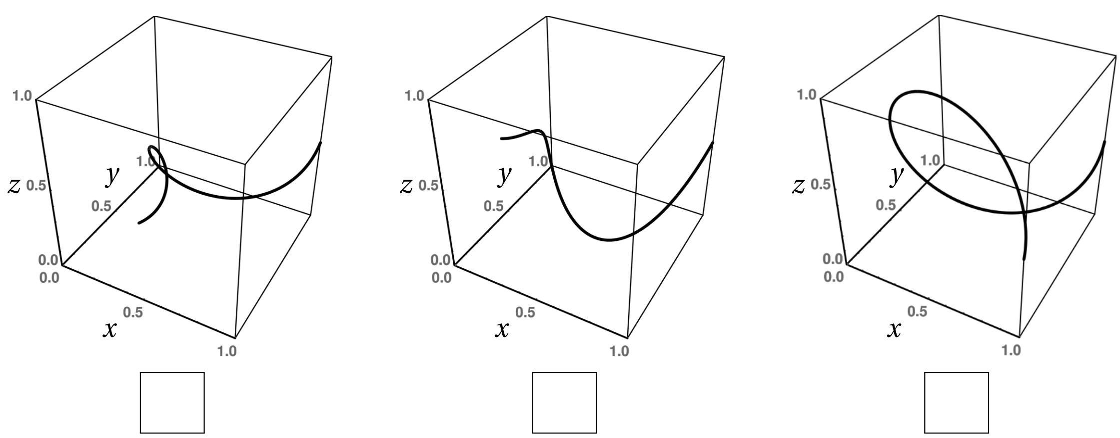
(d) Find $\int_C \nabla g \cdot d\mathbf{r}$:

−12	−9	−6	−3	0	3	6	9	12

6. Consider the curve C in $\mathbb{R}^3$ whose projections onto the xy, xz and yz planes are:



Check the box below the three-dimensional plot of C. **(2 points)**



7. Let C be the curve in three-dimensional space parametrized by $\mathbf{r}(t) = \langle 2\cos t,\ 2\sin t,\ t\rangle$ for $-\pi \le t \le \pi$.

(a) Find the mass of a thin wire in the shape of C, if the density function is $\rho(x, y, z) = x + z + 10$. **(5 points)**

Mass =

(b) Suppose that a particle moves along C starting at $\mathbf{r}(-\pi) = (-2, 0, -\pi)$ and ending at $\mathbf{r}(\pi) = (-2, 0, \pi)$. Find the work done on the particle by the force $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j}$. **(5 points)**

Work =

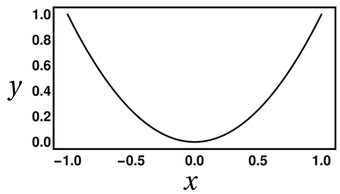
8. (a) Let S_1 be the surface defined by $x = 2z^2 + y^2$ (an elliptic paraboloid). Find an equation for the tangent plane to S_1 at the point $(3, -1, 1)$. **(4 points)**

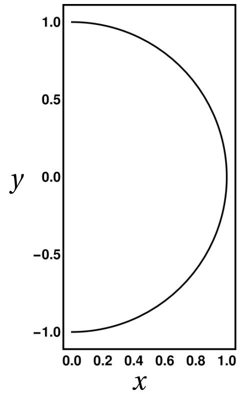
Equation: $\square\, x + \square\, y + \square\, z = \square$

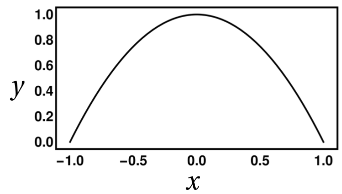
(b) Let S_1 be as in the previous part, and let S_2 be the surface defined by $y^2 + \dfrac{z^2}{4} = 1$ (a cylinder over an ellipse). Find a vector function $\mathbf{r}(t)$ that parametrizes the curve that is the intersection of the surfaces S_1 and S_2. Specify the range of the parameter values so that the function traces the curve exactly once. **(4 points)**

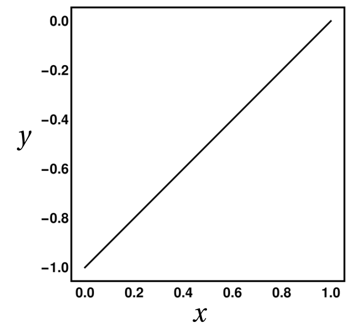
$\mathbf{r}(t) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$ for $\square \le t \le \square$

9. Check the box below the picture of the curve $\mathbf{r}(t) = \langle \sin t,\ \cos^2 t \rangle$, $0 \le t \le 2\pi$. **(2 points)**









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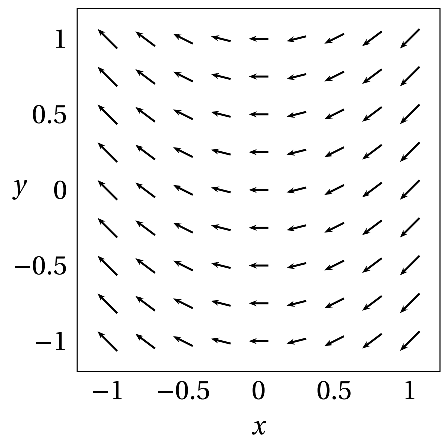
10. A vector field $\mathbf{G}$ is plotted at right.



(a) Circle the formula for $\mathbf{G}$. **(1 point)**

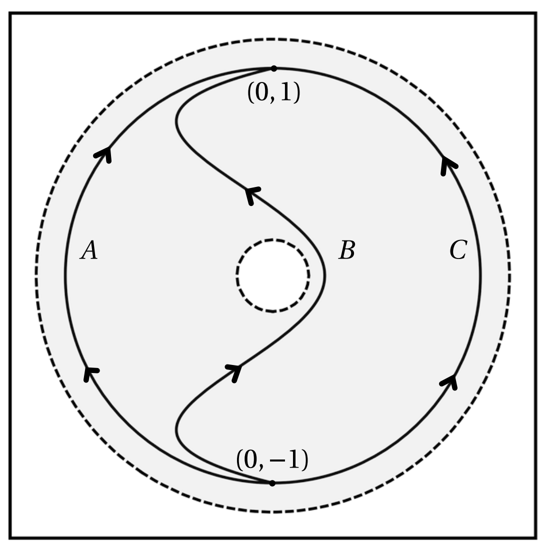
$x\mathbf{i} + y\mathbf{j}$	$-x\mathbf{i} - \mathbf{j}$	$-\mathbf{i} - x\mathbf{j}$	$y\mathbf{i} - \mathbf{j}$

(b) $\mathbf{G}$ is conservative. **(1 point)**

True	False

11. The region D defined by $\{0.03 < x^2 + y^2 < 1.3\}$ is shown at right. Within this region are three curves A, B, C. Each curve starts at $(0,-1)$ and ends at $(0,1)$. Suppose that $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a differentiable vector field defined on D with the properties

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \qquad \int_A \mathbf{F} \cdot d\mathbf{r} = -1, \quad \text{and} \quad \int_C \mathbf{F} \cdot d\mathbf{r} = 2.$$



(a) The region D is simply connected. **(1 point)**

True	False

(b) $\mathbf{F}$ is conservative. **(1 point)**

Yes	No	Cannot determine

(c) Find $\int_B \mathbf{F} \cdot d\mathbf{r}$. **(1 point)**

-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3