1. Find the maximum and minimum value of the function $f(x, y, z) = 3x + y$ on the ellipsoid $3x^2 + 2y^2 + z^2 = 14$. **(5 points)**

Maximum value =

Minimum value =

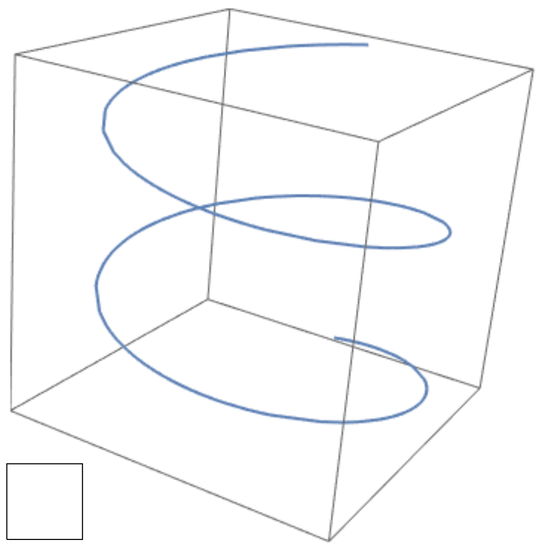
2. Find the length of the curve C parameterized by $\mathbf{r}(t) = \langle 2t, \cos t, \sin t \rangle$ for $0 \le t \le 5\pi$. **(4 points)**

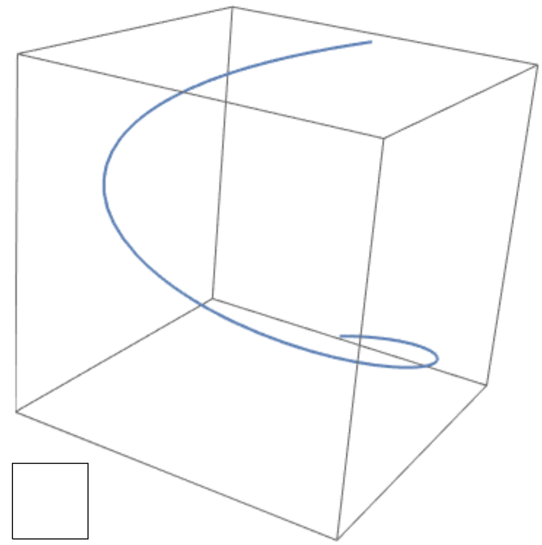
Length = [ ]

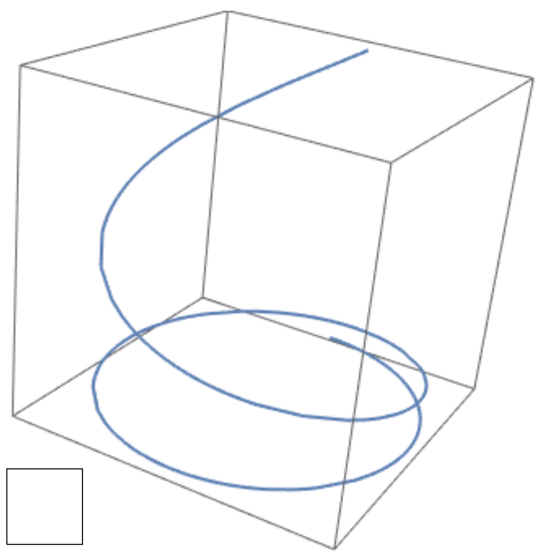
3. Parameterize the curve given by the intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$. Specify the domain (the values of the parameter t) so that the function traces the curve exactly once. **(4 points)**

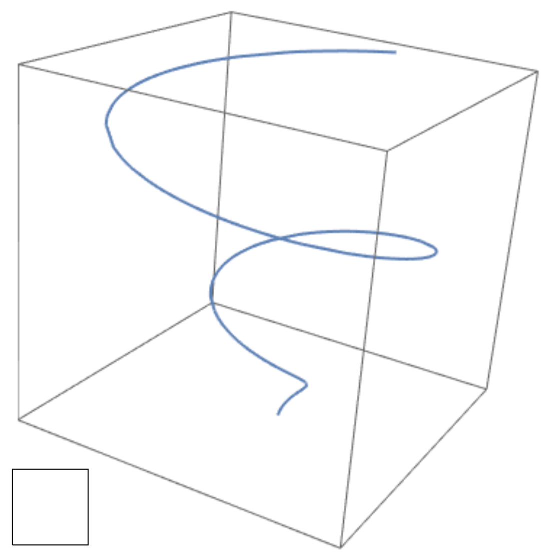
$\mathbf{r}(t) = \langle \quad , \quad , \quad \rangle$ for t in [ ]

4. Let C be the curve parameterized by $\mathbf{r}(t) = \langle \sin(t^2), \cos(t^2), t^2 \rangle$ for $0 \le t \le 2\sqrt{\pi}$. Check the box below the picture of the curve C. **(2 points)**









5. Consider the following domains (subsets) in $\mathbb{R}^2$. For each, circle the characteristics that accurately describe the set; **circle zero, one, or both options**, as appropriate. **(3 points)**

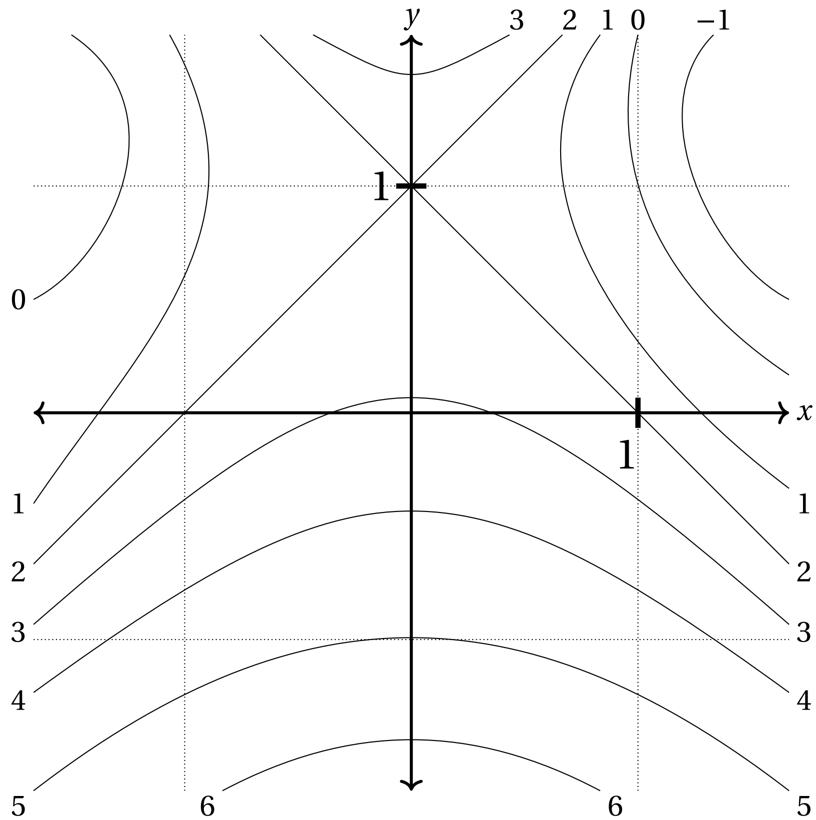
(a) $D_1 = \{(x, y) \mid y^2 \ge x^2 + 1\}$. D_1 is | simply connected closed |

(b) $D_2 = \{(x, y) \mid 1 < x^2 + y^2 \le 2\}$. D_2 is | open bounded |

(c) $D_3 = \{(x, y) \mid (x, y) \ne (0, 0)\}$. D_3 is | simply connected bounded |

6. The contour map of a differentiable function $f(x,y)$ is shown below.

Circle the *best response* for each of parts (a) - (d) below.



(a) **(2 points)** $\nabla f(-1,0) =$

$\langle 0,1\rangle$	$\langle -1,2\rangle$	$\langle 1,-1\rangle$	$\langle 2,-2\rangle$
$\langle 1,0\rangle$	$\langle 1,-2\rangle$	$\langle 0,-2\rangle$	$\langle -2,2\rangle$

(b) **(2 points)** The maximum rate of change of f at $(0,-1)$ is

1	2	3	4	5	6

(c) **(2 points)** $D_{\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle} f(1,0) =$

-2	-1	0	1	2

(d) **(2 points)** If C is the straight line segment from $(0,-1)$ to $(1,0)$, then

$\frac{1}{\sqrt{2}}\int_C f\,ds =$

-5	$-\frac{7}{2}$	$-\frac{3}{2}$	0	$\frac{1}{2}$	2	$\frac{7}{2}$	5

Scratch Space

7. Three continuous vector fields, **F**, **G**, **H** on the plane are plotted below in the region with $-1 < x < 1$ and $-1 < y < 1$.

Circle the *best response* for each of the following.

(a) **(2 points)** Which vector field is ∇f, where $f(x, y) = xy$?

F	**G**	**H**

(b) **(4 points)** A particle moves along a straight line from $(-1, -1)$ to $(1, 1)$. If the vector fields represent force fields, then:

The work done by **G** on the particle is...

positive	negative	zero

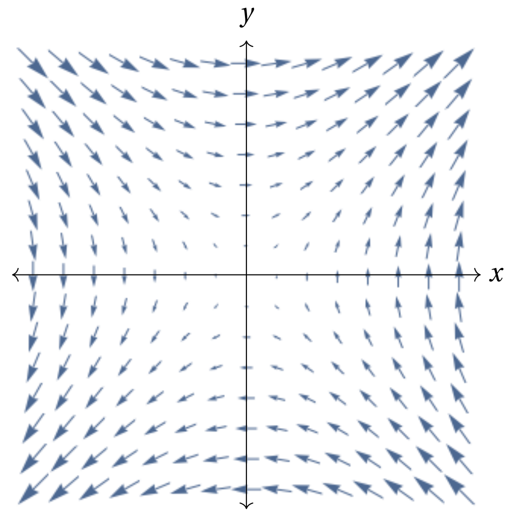
.

The work done by **H** on the particle is...

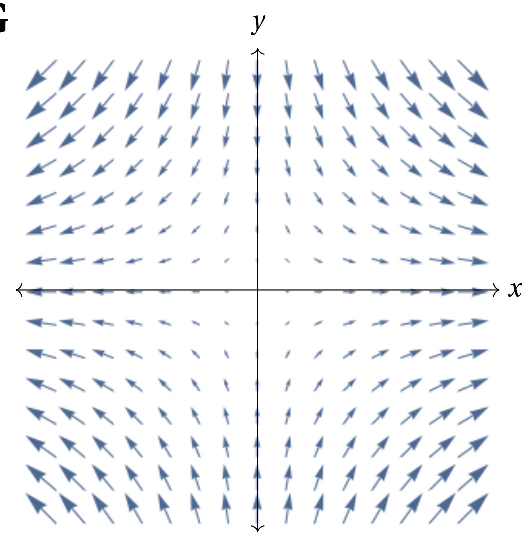
positive	negative	zero

.

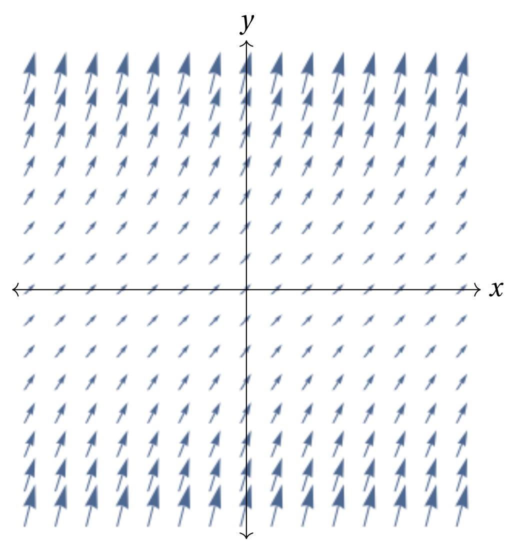
F



G



H



8. Consider the vector field $\mathbf{F}(x,y) = \langle \frac{1}{y}e^{\frac{x}{y}}, -\frac{x}{y^2}e^{\frac{x}{y}} + 2y\rangle$. Let C be the straight line segment from $P = (1,1)$ to $Q = (2,2)$ parametrized by

$$\mathbf{r}(t) = \langle t, t\rangle, \quad \text{for} \quad 1 \le t \le 2.$$

(a) Use the definition of the line integral of a vector field along a curve to compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ directly, using the above parametrization. (**No credit will be given for computations using any other method.**)

(5 points)

$\int_C \mathbf{F}\cdot d\mathbf{r} =$ ☐

(b) The vector field $\mathbf{F}$ is conservative. Find a function f such that $\nabla f = \mathbf{F}$. **(2 points)**

$f(x,y) =$ ☐

(c) Use your work from part (b) to check your result from part (a). (Show your work and explain your method. Note: If you were not able to solve part (a) or part (b), explain how you *could* check your answer assuming that you had found a number N in part (a) and a function f in part (b).) **(2 points)**

9. Suppose that $f(x, y)$ is a differentiable function with continuous second order partial derivatives and values given by the table below. **(5 points)**

(x, y)	$f(x, y)$	$f_x(x, y)$	$f_y(x, y)$	$f_{xx}(x, y)$	$f_{yy}(x, y)$	$f_{xy}(x, y)$
$(2, 1)$	0	0	-1	2	3	1
$(0, 1)$	-1	0	0	-2	-2	-2
$(1, 0)$	2	0	0	2	1	1

For each of the given points, circle the best description of the point.

$(2, 1)$	not critical	local minimum	local maximum	saddle point	undetermined
$(0, 1)$	not critical	local minimum	local maximum	saddle point	undetermined
$(1, 0)$	not critical	local minimum	local maximum	saddle point	undetermined

10. Let D be the set of all points (x, y) in $\mathbb{R}^2$ except for $(0, 0)$. In each part below, indicate whether a continuous vector field with domain D and the property described is *necessarily conservative* or *not necessarily conservative*. (If the vector field is never conservative, circle *not necessarily conservative*.) **(4 points)**

(a) $\mathbf{F}_1(x, y) = \langle -y, x \rangle$.

$\mathbf{F}_1$ is

necessarily conservative	not necessarily conservative

(b) $\mathbf{F}_2$ has the path independence property; that is, the line integral $\int_C \mathbf{F}_2 \cdot d\mathbf{r}$ is independent of path in D.

$\mathbf{F}_4$ is

necessarily conservative	not necessarily conservative

(c) $\mathbf{F}_3 = \langle P, Q \rangle$ where $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ over D.

$\mathbf{F}_3$ is

necessarily conservative	not necessarily conservative

(d) $\mathbf{F}_4$ has the property that for the unit circle $C_1 = \{x^2 + y^2 = 1\}$, $\int_{C_1} \mathbf{F}_4 \cdot d\mathbf{r} = 0$.

$\mathbf{F}_4$ is

necessarily conservative	not necessarily conservative