1. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x - y$. Let C be the circle $x^2 + y^2 = 2$.

(a) Use Lagrange multipliers to find the absolute max and min of f on C. **(5 points)**

Absolute max of f on C is [ ] which occurs at the point(s) [ ]

Absolute min of f on C is [ ] which occurs at the point(s) [ ]

(b) Find the absolute max of f on $\mathbb{R}^2$ if it exists. **(1 point)**

Absolute max of f on $\mathbb{R}^2$ is [ ]

2. Consider the differentiable function $f(x,y)$ on the rectangle $D = \{0 \le x \le 6 \text{ and } 0 \le y \le 4\}$ whose contours are shown below right. For each part, circle the best answer. **(1 point each)**

(a) The maximum value of f on D is:

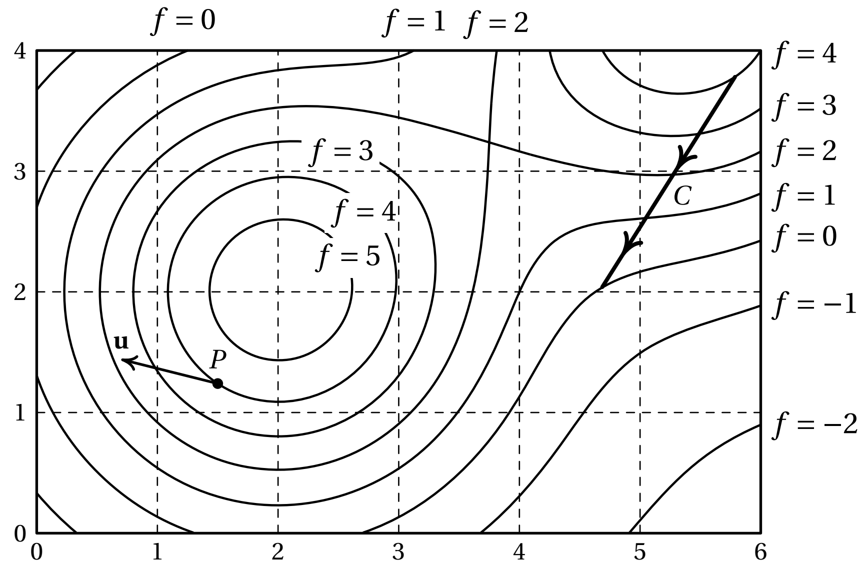
0	3	6	9	DNE

(b) The value of $D_{\mathbf{u}} f(P)$ is:

negative	zero	positive

(c) The number of critical points of f in D which are saddles is:

0	1	2	3



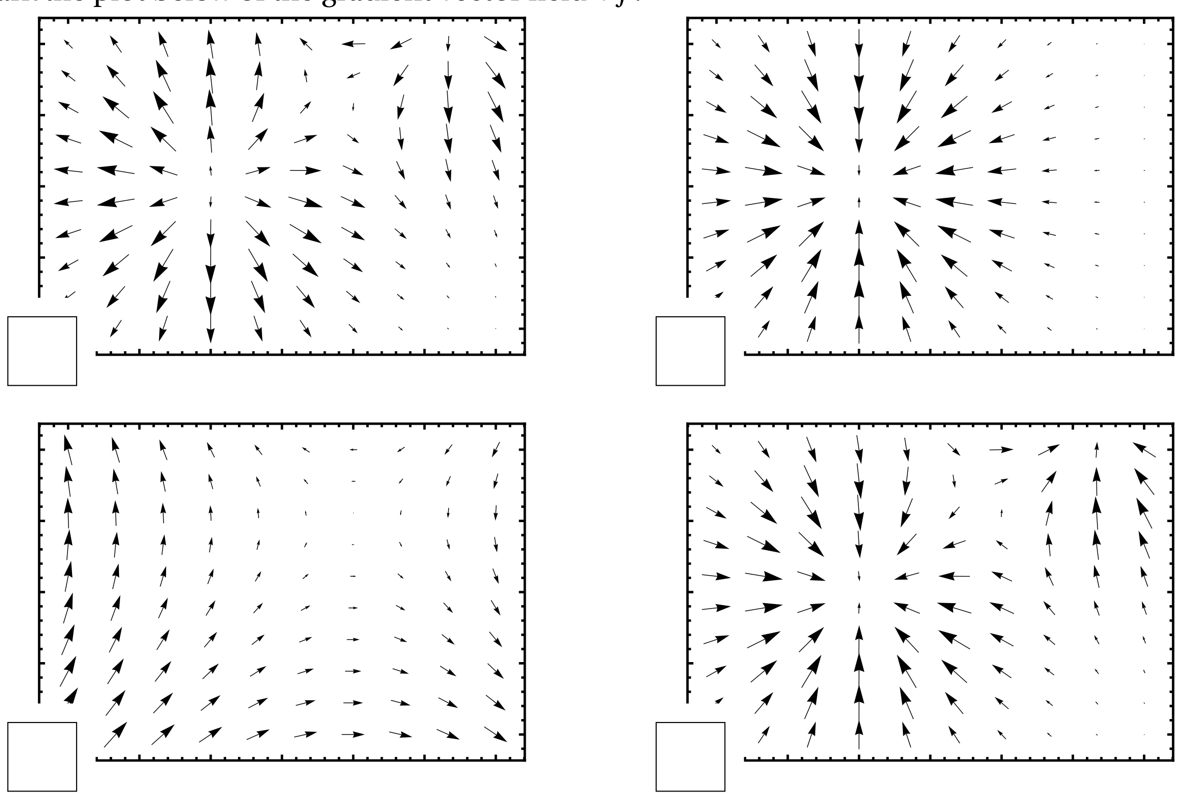
(e) The integral $\int_C f \, ds$ is:

−8	−4	−2	0	2	4	8

(f) The integral $\int_C \nabla f \cdot dr$ is:

−6	−4	−2	0	2	4	6

(h) Mark the plot below of the gradient vector field ∇f.



3. Let C be the curve in $\mathbb{R}^3$ that is the intersection between the circular cylinder $x^2 + (z-1)^2 = 4$ and the plane $x + y + z = 1$.

(a) Find a vector function $\mathbf{r}(t)$ that parameterizes C, traversing the whole curve exactly once. Be sure to specify the domain of your parameterization. **(4 points)**

$\mathbf{r}(t) = \left\langle \quad , \quad , \quad \right\rangle$ for $\quad \leq t \leq \quad$

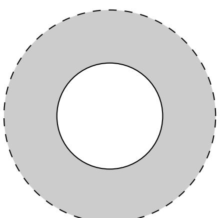
(b) The vector $\mathbf{v} = \langle 1, 2, 3 \rangle$ is tangent to C at some point: true false **(1 point)**

Scratch Space

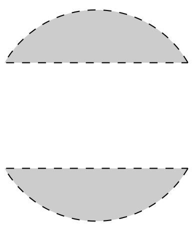
4. The vector field $\mathbf{F}(x, y) = \langle\, y^2 + 1,\ 2xy + 1 \,\rangle$ on $\mathbb{R}^2$ is conservative. Find a function $f(x, y)$ where $\mathbf{F} = \nabla f$. **(3 points)**

$f(x, y) =$

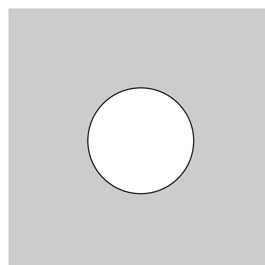
5. Consider the following four regions in the plane: **(1 point each)**



$R_1 = \{1 \le x^2 + y^2 < 4\}$

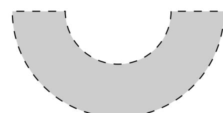


$R_2 = \left\{ \begin{matrix} x^2 + y^2 < 4 \\ \text{and } |y| > 1 \end{matrix} \right\}$



$R_3 = \{4 \le x^2 + y^2\}$

$R_4 = \left\{ \begin{matrix} 1 < x^2 + y^2 < 4 \\ \text{and } y < 0 \end{matrix} \right\}$



(a) Which region is closed? R_1 R_2 R_3 R_4

(b) Which region is simply connected (and thus also connected)? R_1 R_2 R_3 R_4

Scratch Space

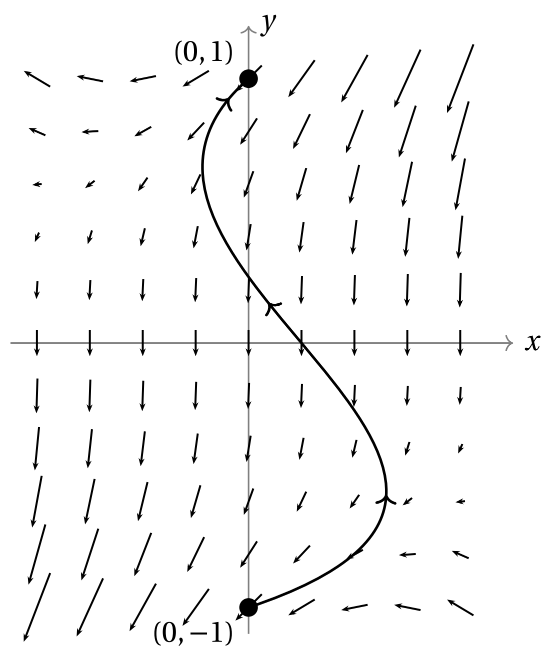
6. Find the mass of a thin wire in the shape of the curve parameterized by $\mathbf{r}(t) = \langle \sin t,\ 2t,\ \cos t \rangle$ for $0 \le t \le \pi$, if the wire has density function $\rho(x, y, z) = y$. **(4 points)**

Mass =

7. A vector field $\mathbf{F}$ is shown at right; for scale, $\mathbf{F}(0,0) = \langle 0,\ -0.1 \rangle$. Assuming that $\mathbf{F}$ is conservative, circle the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve shown from $(0, -1)$ to $(0, 1)$.

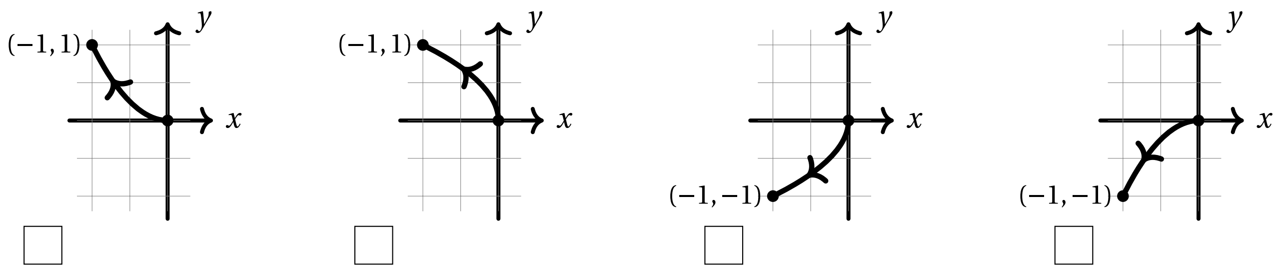
−0.3	−0.2	−0.1	0	0.1	0.2	0.3

(2 points)



8. Let C be the curve in $\mathbb{R}^2$ parameterized by $\mathbf{r}(t) = \langle -t^2,\ t \rangle$ for $0 \le t \le 1$.

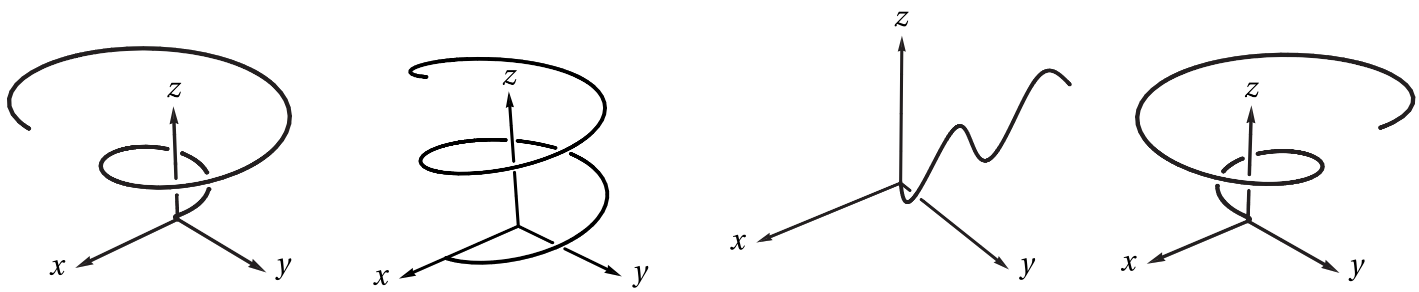
(a) Mark the picture of C from among the choices below. **(1 point)**



(b) For the vector field $\mathbf{F} = \langle 1,\ x \rangle$, calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(4 points)**

$\int_C \mathbf{F} \cdot d\mathbf{r} =$

9. Mark the picture of the curve in $\mathbb{R}^3$ parameterized by $\mathbf{r}(t) = \langle t\sin t,\ t\cos t,\ t\rangle$ for $0 \le t \le 4\pi$. **(2 points)**



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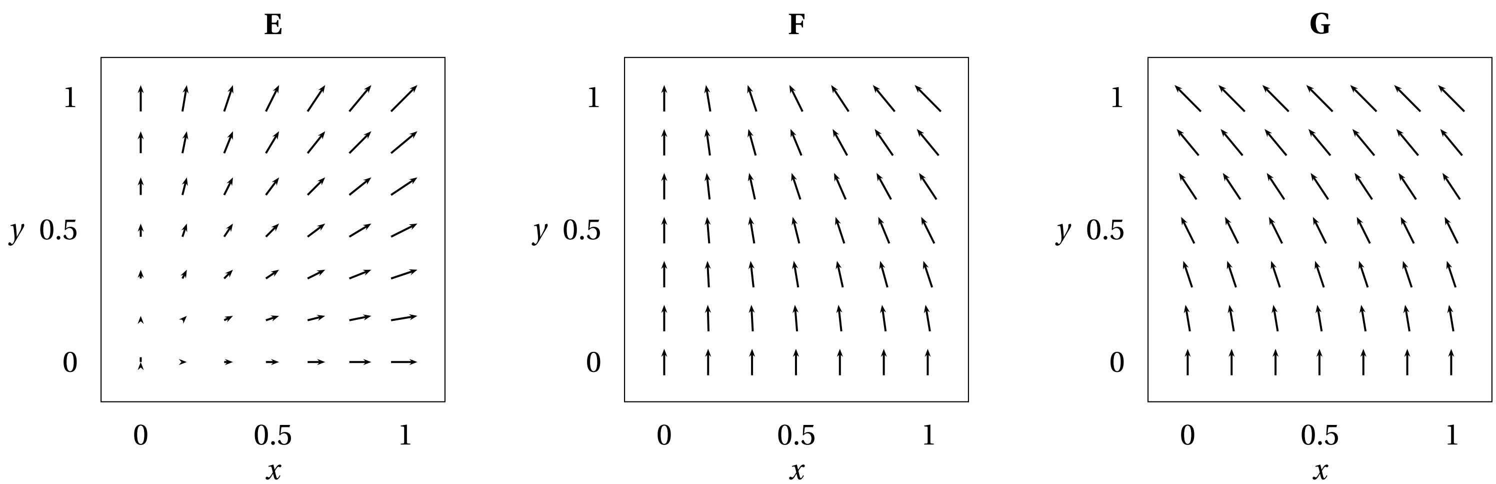
10. Consider the three vector fields **E**, **F**, and **G** on $\mathbb{R}^2$ shown below. **(1 point each)**



(a) One of these vector fields is $\langle -xy,\ 1\rangle$. Circle its name here: **E F G** **(1 point)**

(b) Exactly one of these vector fields is conservative. Circle it here: **E F G** **(1 point)**

(c) Exactly one of the following is a flowline (also called a streamline or integral curve) for **E** parameterized by time for $0 \le t \le 1$. Circle it. **(1 point)**

$\mathbf{r}(t) = \langle t,\ 1-t\rangle$	$\mathbf{r}(t) = \langle t,\ \sqrt{t}\rangle$	$\mathbf{r}(t) = \langle t,\ t\rangle$	$\mathbf{r}(t) = \langle e^t,\ e^t\rangle$

Scratch Space