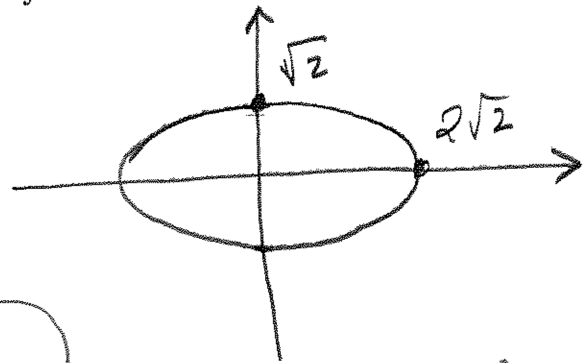
1. (6 points) Find the points on the curve $\underbrace{x^2+4y^2}_{g(x,y)} = 8$ where the function $f(x,y) = -x+2y$ attains its maximum *max* and minimum *min*, and say what *max* and *min* are.

Use Lagrange:




$\nabla f = \langle -1, 2\rangle = \lambda \nabla g$
$= \lambda\langle 2x, 8y\rangle$

So we solve: $\begin{cases} -1 = \lambda 2x \\ 2 = \lambda 8y \end{cases}$ and $x^2+4y^2=8$

Constraint curve is an ellipse.

Now $\lambda \neq 0$ since $\lambda = 0$ leads to $-1=0$ and $2=0$.
So, solving both eqns for $\frac{1}{\lambda}$ gives $\frac{1}{\lambda} = -2x$ and $\frac{1}{\lambda} = 4y$. Thus $-2x = 4y$, that is $x = -2y$. Then $8 = x^2+4y^2 = 4y^2+4y^2 = 8y^2$ and so $y^2 = 1 \Rightarrow y = \pm 1$. So there are 2 critical points: $(-2, 1)$ and $(2,-1)$. As the ellipse $x^2+4y^2=8$ is closed and bounded, the Extreme Value Theorem tells us that absolute min/max exist.

Hence

*max* = $\boxed{4}$ at the point(s) $\boxed{(-2, 1)}$

*min* = $\boxed{-4}$ at the point(s) $\boxed{(2,-1)}$

2. (3 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function with continuous second order partial derivatives at every point. Assume that $f(0,0) = 1$, $f_x(0,0) = 0$, $f_y(0,0) = 0$, $f_{xx}(0,0) = 5$, $f_{xy}(0,0) = 2$, $f_{yx}(0,0) = 2$, $f_{yy}(0,0) = -1$. Determine whether the point $(0,0)$ is critical and, if so, say whether it is a local minimum, a local maximum, or a saddle point for $f$. Circle your answer.

As $\nabla f(0,0) = (f_x(0,0), f_y(0,0)) = (0,0)$ we see $f$ is a critical pt. Applying the $2^{nd}$ derivative test gives $D = \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = -9$ which means:

| | | |
|---|---|---|
| Not a critical point | A local min | A local max |
| A saddle | We do not have enough information | |

3. (3 points) Let $D$ be the set of points $(x, y)$ in $\mathbb{R}^2$ such that $1 < x^2 + y^2 < 4$. Which of the following are properties of $D$? Circle all that apply.

| | | |
|---|---|---|
| open | connected | simply connected |

4. (6 points) The vector field $\mathbf{F}(x,y,z) = \langle yz, xz, xy+2z\rangle$ is conservative. Find a function $f$ so that $\nabla f = \mathbf{F}$. **(No partial credit: You can check your answer!)**

Want $\frac{\partial f}{\partial x} = yz \Rightarrow f = \int yz\,dx = xyz + C(y,z)$

Then $\frac{\partial f}{\partial y} = xz + \frac{\partial C(y,z)}{\partial y}$ and since we want this to be $xz$ we just make $C$ depend only on $z$.

Then $\frac{\partial f}{\partial z} = xy + \frac{\partial C(z)}{\partial z} \Rightarrow \frac{\partial C(z)}{\partial z} = 2z \Rightarrow C(z) = z^2$.

Thus, we can take $f = xyz + z^2$

$f(x,y,z) = \boxed{xyz + z^2}$

5. (6 points) Consider the vector field $\mathbf{F}(x,y) = \langle e^{(x^2)}, \sin(y)\rangle$. Is $\mathbf{F}$ conservative? Circle the correct response

P Q

| | | |
|---|---|---|
| Yes | No | We do not have enough information. |

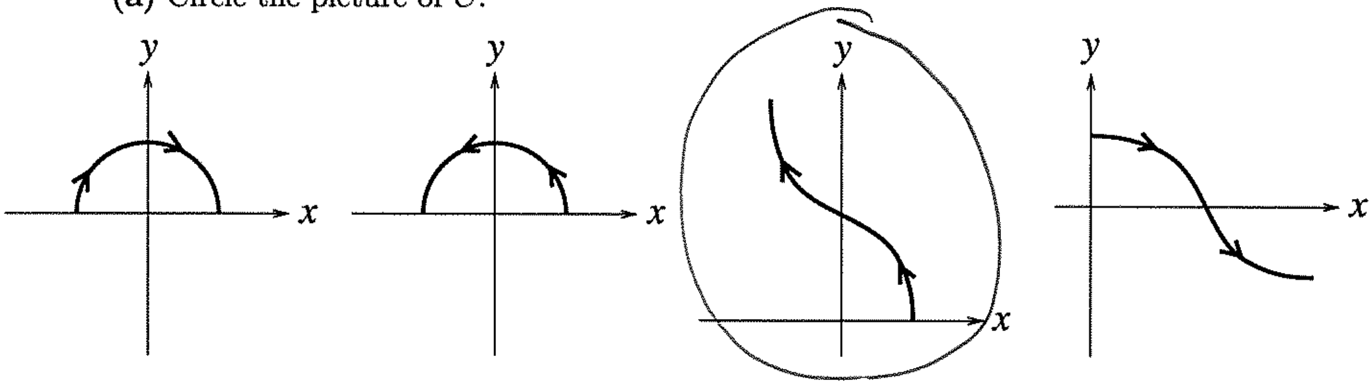
**and justify your answer.**

$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} e^{x^2} = 0$ and $\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} \sin(y) = 0$

Since $\vec{F}$ is defined on all of $\mathbb{R}^2$, which is open and simply connected, then this means $\vec{F}$ is conservative.

6. (7 points) Consider the oriented curve $C$ parameterized by $\mathbf{r}(t) = \langle \cos(t), t \rangle$, $t \in [0, \pi]$.

**(a)** Circle the picture of $C$.




r(t) is describing part of the curve x = cos(y)

**(b)** Calculate the integral $\int_C \langle 1, y^2 + x \rangle \cdot d\mathbf{r}$.

$$\vec{r}'(t) = \langle -\sin t, 1 \rangle$$

$$\int_C \langle 1, y^2 + x \rangle \cdot d\vec{r} = \int_0^{\pi} \langle 1, t^2 + \cos t \rangle \cdot \langle -\sin t, 1 \rangle \, dt$$

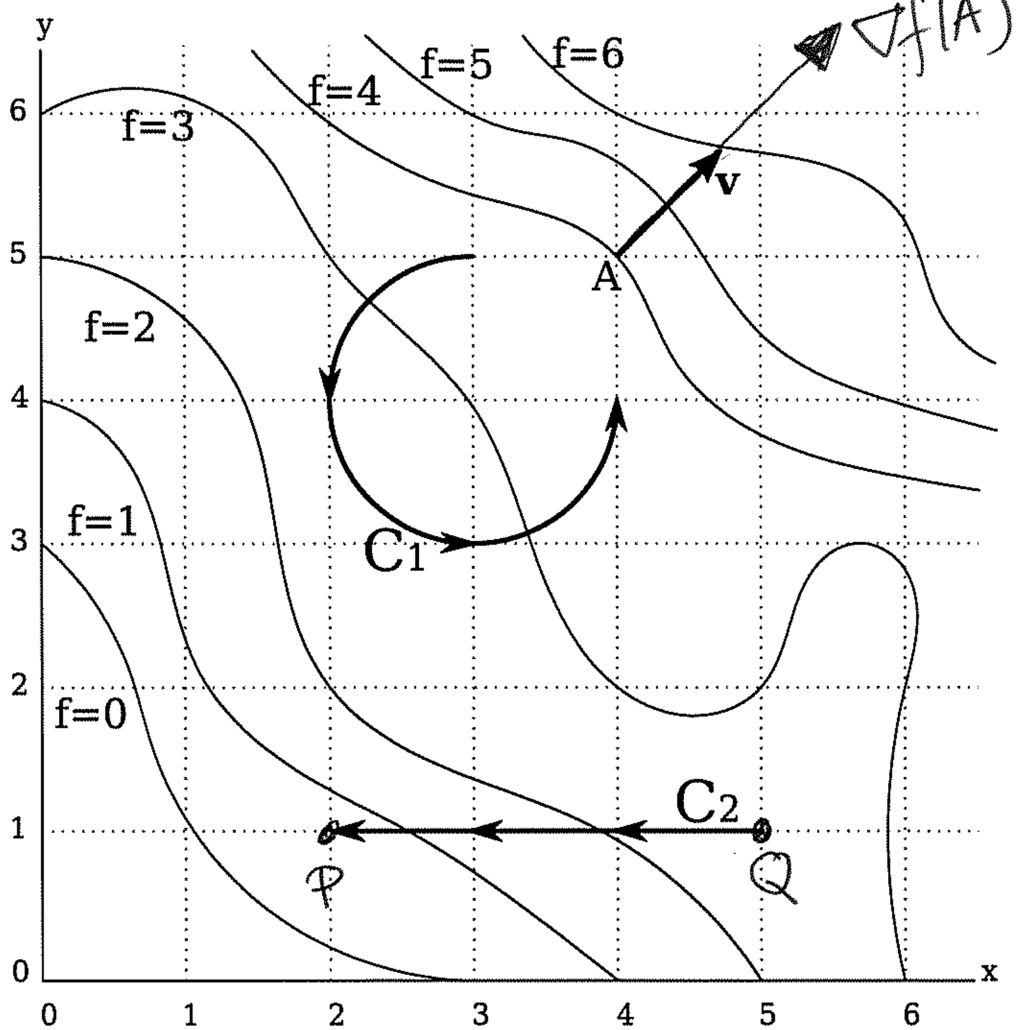
$$= \int_0^{\pi} -\sin t + t^2 + \cos t \, dt$$

$$= \cos t + t^3/3 + \sin t \Big|_0^{\pi} = (-1 + \pi^3/3 + 0) - (1 + 0^3/3 + 0)$$

$$= \pi^3/3 - 2$$

$\int_C \langle 1, y^2 + x \rangle \cdot d\mathbf{r} =$ $\boxed{\pi^3/3 - 2}$

7. (7 points) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ whose contour diagram is shown below, as well as the curves $C_1$, $C_2$, the point $A$, and the unit vector $\mathbf{v}$. For each part circle the best answer




(a) The sign of $\nabla f(A) \cdot \mathbf{v}$ is

| | | |
|---|---|---|
| negative | zero | positive |

**This computes the direcitonal derivative $D_v f(A)$**

**and the function is increasing in the direction v.**

(b) The value of $\int_{C_1} f \, ds$ is

| | | | | |
|---|---|---|---|---|
| -14 | -4 | 4 | 14 | 24 |

$C_1$ has len $\approx 4.5$, average value of $f$ on $C_1 \approx 3$.

(c) The sign of $\int_{C_2} \nabla f \cdot d\mathbf{r}$ is

| | | |
|---|---|---|
| negative | zero | positive |

$\hookrightarrow f(P) - f(Q) \approx 0.7 - 2.1 \approx -1.4$