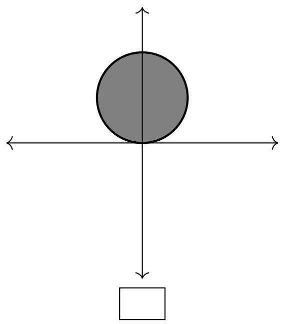
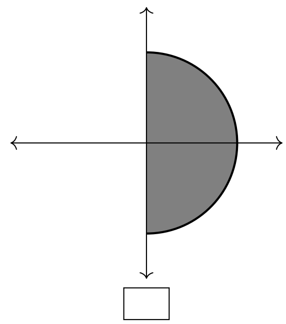
1. Suppose the integral of the function $f(x, y) = x^2 + y$ over a region R has the following form after changing to polar coordinates:

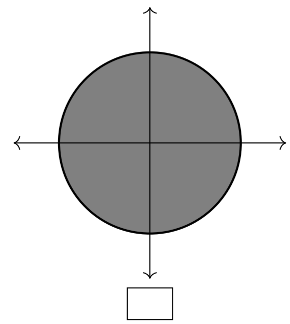
$$\iint_R x^2 + y \, dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} g(r,\theta) \, dr \, d\theta,$$

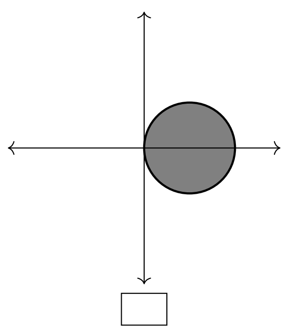
for some function $g(r,\theta)$.

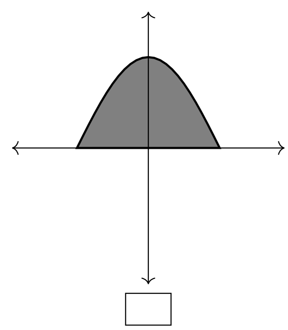
(a) Which of the following shows the region R in the xy-plane? **(2 points)**











(b) Find the integrand $g(r,\theta)$ and fill in blank in the integral below. **(3 points)**

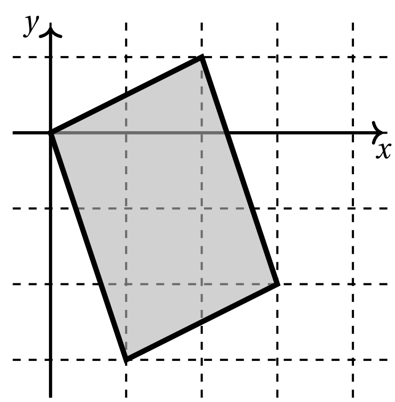
$$\iint_R x^2 + y \, dA = \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \qquad\qquad\qquad dr \, d\theta}$$

2. Compute the following double integral: **(5 points)**

$$\int_1^2 \int_0^x 2x + 2y \; dydx.$$

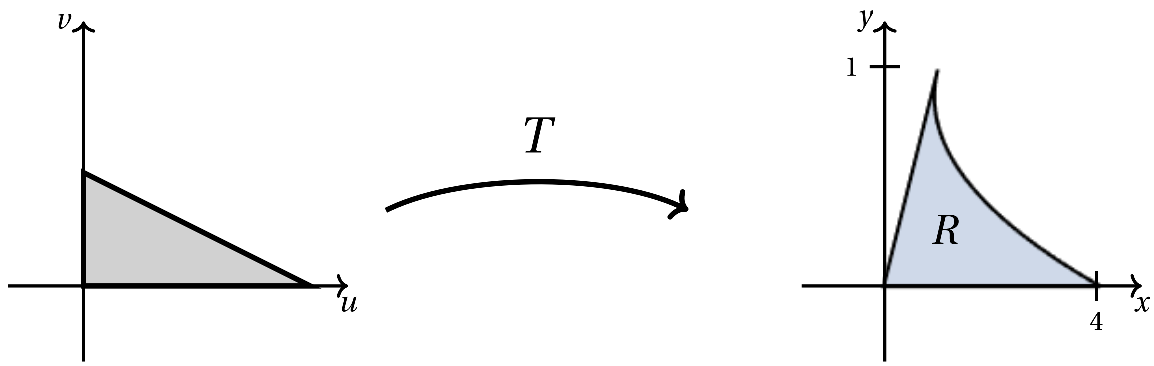
$$\int_1^2 \int_0^x 2x + 2y \; dydx = \boxed{}$$

3. Find a linear transformation T that sends the unit square $[0,1] \times [0,1]$ to the parallelogram drawn to the right against a dashed grid of unit squares. **(3 points)**



$$T(u, v) = \boxed{\Big(\qquad\qquad , \qquad\qquad \Big)}$$

4. The transformation $T(u,v) = (u^2 + v, v)$ takes the triangle with vertices $(0,0)$, $(2,0)$, and $(0,1)$ to the region R as shown below. Find the missing limits of integration and circle the correct integrand for the iterated integral that computes *the area of* R. (Note that the order of integration is already determined.) **(5 points)**

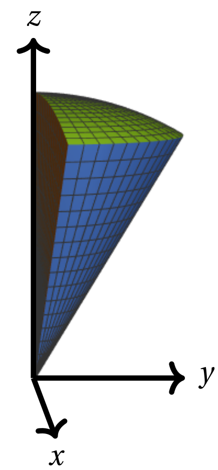


Fill in the limits of integration

$$\text{Area}(R) = \int \int \quad g(u,v)\, dv\, du$$

Circle the correct integrand. $g(u,v) =$ | $u^2 v$ | uv^2 | $2u$ | $2v$ | $u^2 + v$ | $v^2 + u$ |

5. In this problem you are to set up the integral of the function yz over the region R (shown to the right) in the first octant above the cone $z^2 = 3x^2 + 3y^2$ and inside the sphere $x^2 + y^2 + z^2 = 16$. Check the box next to the integral with the correct limits of integration, then circle the correct integrand. **(4 points)**



$$\iiint_R yz\,dV =$$

☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}}\int_0^{2} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$ ☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_0^{2} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$

☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}}\int_0^{4} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$ ☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_0^{4} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$

☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{6}}\int_0^{16} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$ ☐ $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}}\int_0^{16} h(\rho,\theta,\phi)\,d\rho\,d\phi\,d\theta$

$h(\rho,\theta,\phi) =$

$\rho^3\sin\theta\sin^2\phi\cos\phi$	$\rho^4\sin\theta\sin^2\phi\cos\phi$	$\rho^5\sin\theta\cos^2\phi\sin\phi$
$\rho^3\cos\theta\sin^2\phi\cos\phi$	$\rho^4\cos\theta\sin^2\phi\cos\phi$	$\rho^5\cos\theta\cos^2\phi\sin\phi$

6. Consider a solid object with density ρ_0 that occupies a region E in $\mathbf{R}^3$. The moment of inertia of the solid around the x-axis is given by

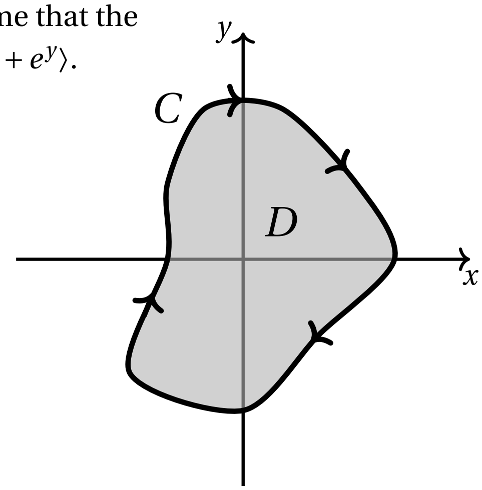
$$I_x = \iiint_E g(x,y,z)\,dV,$$

for some function $g(x,y,z)$. Circle the correct integrand. **(2 points)**

$g(x,y,z) =$

$\rho_0(x^2+y^2+z^2)$	$\rho_0(y^2+z^2)$	$\rho_0(x^2+y^2)$	$\rho_0(x^2+z^2)$	$\rho_0 x^2$	$\rho_0 x$	$\rho_0 y^2$	$\rho_0 y$

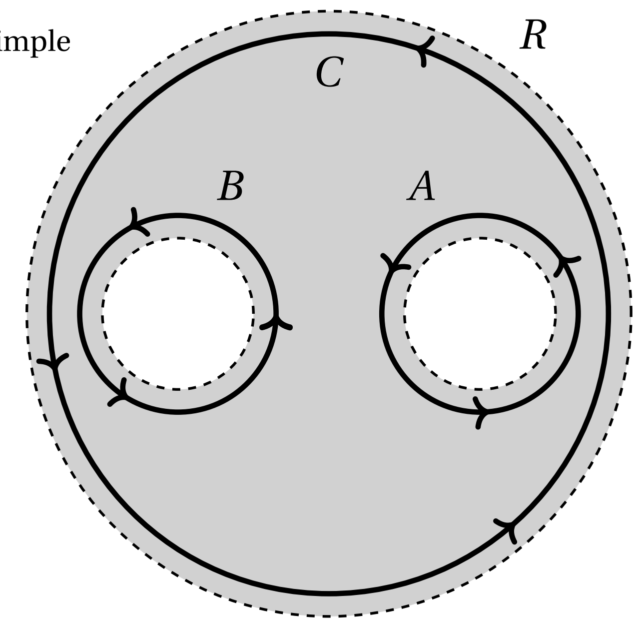
7. Let C be the oriented curve shown at right, oriented **clockwise**. Assume that the region D bounded by C has area 6. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle \sin x, 2x + e^y \rangle$.
(4 points)



$\int_C \mathbf{F}\cdot d\mathbf{r} =$ [ ]

8. Consider the region R shown at the right which contains simple closed curves A, B, C, all oriented counterclockwise. Suppose that $\mathbf{F} = \langle P, Q \rangle$ is a vector field with continuous partial derivatives on R with the properties

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}, \quad \int_A \mathbf{F}\cdot d\mathbf{r} = 2 \quad \text{and} \quad \int_B \mathbf{F}\cdot d\mathbf{r} = -1$$



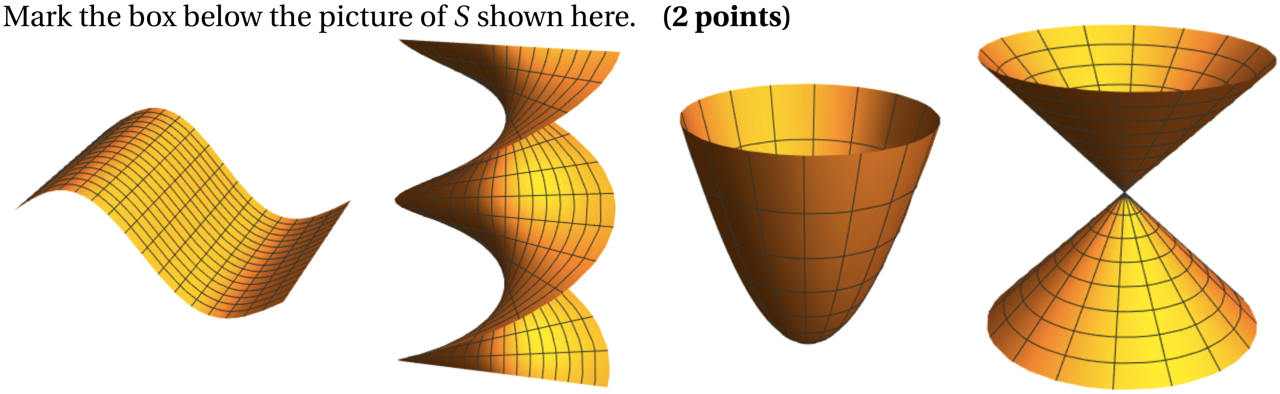
(a) Find $\int_C \mathbf{F}\cdot d\mathbf{r}$. Circle your answer below. **(2 points)**

$\int_C \mathbf{F}\cdot d\mathbf{r} =$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

(b) **True or False?** (Circle your answer.) The vector field $\mathbf{F}$ is conservative. **(2 points)** | True | False |

9. Let S be the surface in $\mathbf{R}^3$ parameterized by $\mathbf{r}(u,v) = \langle v\cos u, v\sin u, u\rangle$, for u in $[0,2\pi]$ and v in $[-2,2]$.

(a) Mark the box below the picture of S shown here. **(2 points)**



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(b) Compute the partial derivatives of $\mathbf{r}$. **(1 point each)**

$\mathbf{r}_u = \Big\langle \quad , \quad , \quad \Big\rangle$

$\mathbf{r}_v = \Big\langle \quad , \quad , \quad \Big\rangle$

(c) Find a vector $\mathbf{n}$ that is perpendicular to the tangent plane to S at the point $\mathbf{r}\left(\frac{\pi}{6},1\right) = \left(\frac{\sqrt{3}}{2},\frac{1}{2},\frac{\pi}{6}\right)$. **(2 points)**

$\mathbf{n} = \Big\langle \quad , \quad , \quad \Big\rangle$

(d) Fill in the limits and integrand of the double integral below that computes $\iint_S 2y\,dS$. Do not evaluate. **(3 points)**

$$\int_S 2y\,dS = \int \int \qquad du\,dv$$

10. Let S be the triangle through the points $(0,0,0)$, $(2,1,0)$, and $(1,0,3)$. Parameterize the surface S by $\mathbf{r}\colon D \to \mathbf{R}^3$, being sure to specify the domain D of the parameterization in the (u,v) plane.
Note: you will receive partial credit if you parameterize the plane through those three points. **(4 points)**

$$D = \left\{ (u,v) : \quad \le u \le \quad , \quad \le v \le \quad \right\}$$

$$\mathbf{r}(t) = \left\langle \quad , \quad , \quad \right\rangle .$$