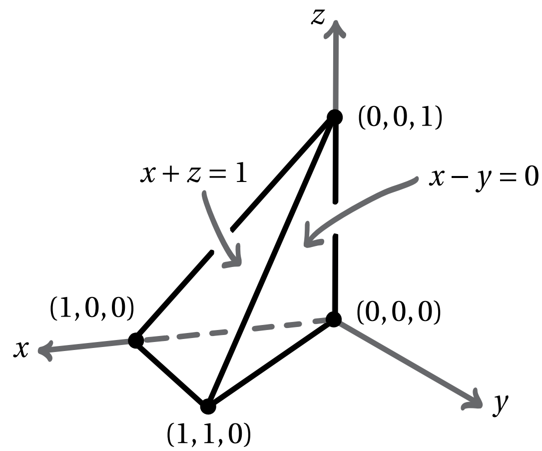
1. Fill in the limits and integrand of the triple integral below so that it computes the volume of the tetrahedron shown at right. **(4 points)**

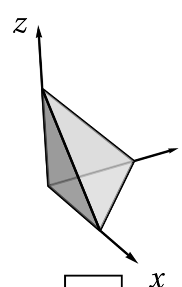


$$\text{Volume} = \int \qquad \int \qquad \int \qquad\qquad d \; d \; d$$

2. Mark the box below the picture corresponding to the region of integration for the triple integral:

$$\int_0^1 \int_z^1 \int_0^{1-y} f(x, y, z)\; dx\, dy\, dz$$

(2 points)



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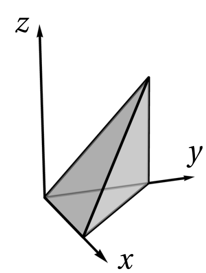

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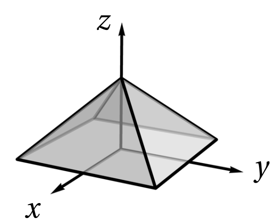

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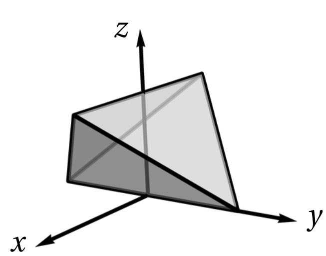

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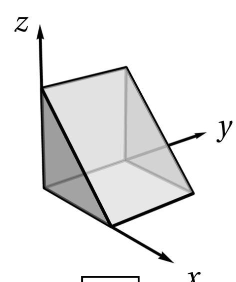

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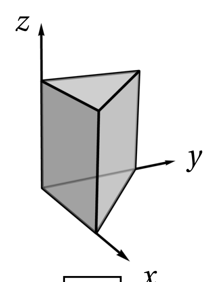

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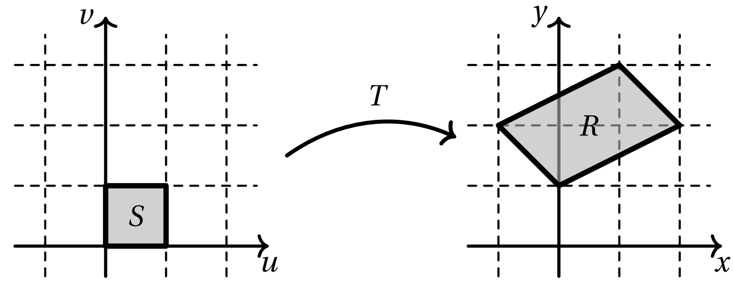
3. Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking the unit square S to the parallelogram R shown at right, where both are shown against a grid of unit squares. **(4 points)**

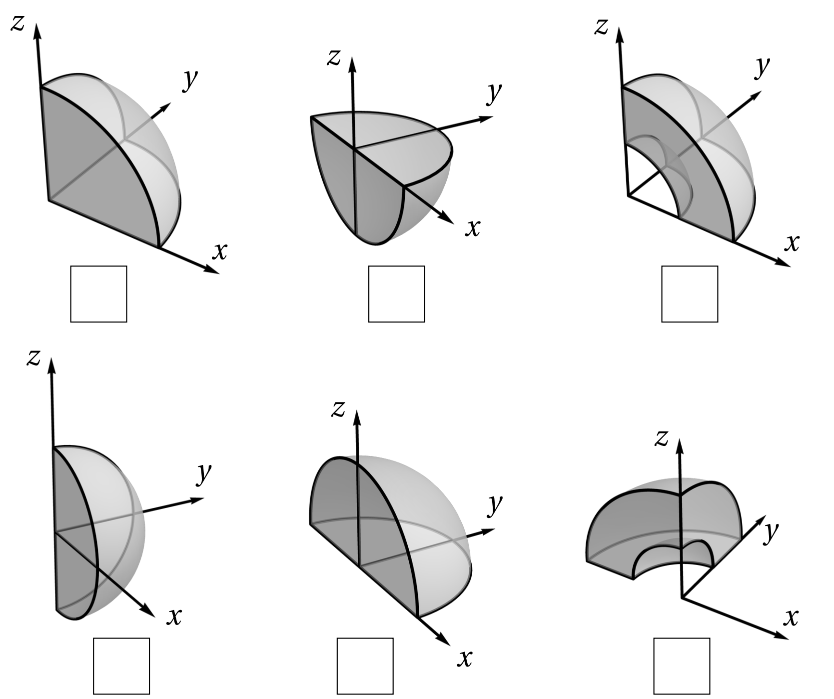


$T(u, v) = (\qquad\qquad , \qquad\qquad)$

4. For each of the given integrals, label the box below the picture of the corresponding region of integration in spherical coordinates. **(2 points each)**

(A) $\displaystyle\int_{\pi/2}^{\pi}\int_{0}^{\pi/2}\int_{1}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

(B) $\displaystyle\int_{0}^{\pi}\int_{\pi/2}^{\pi}\int_{0}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

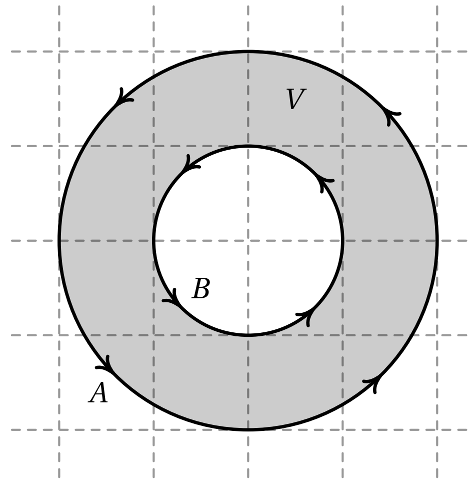


Scratch Space

5. Let A and B be the oriented circles shown at right against a grid of unit squares, and let V be the region between them. Suppose $F = \langle P, Q\rangle$ is a vector field on V where

$$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} - 3 \quad \text{and} \quad \int_A \mathbf{F}\cdot d\mathbf{r} = 10.$$

Compute $\int_B \mathbf{F}\cdot d\mathbf{r}$. **(5 points)**

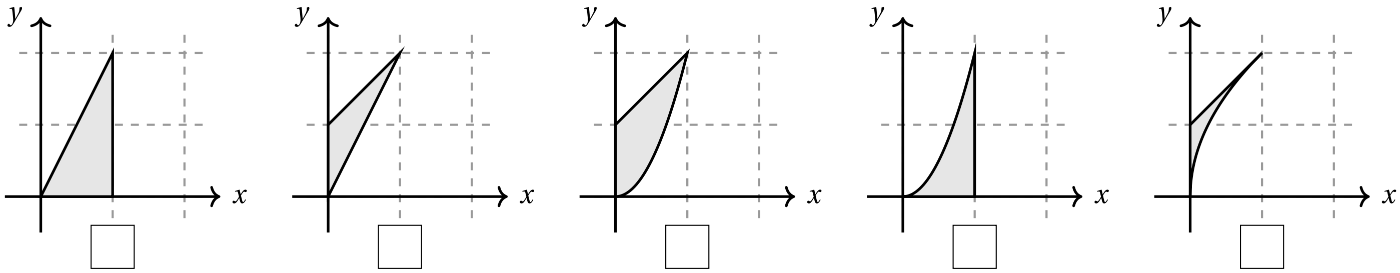


$\int_B \mathbf{F}\cdot d\mathbf{r} =$

Scratch Space

6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation $T(u,v) = (v,\ u - v + 2uv)$. Let S be the triangle in the (u,v)–plane whose vertices are $(0,0), (1,0), (1,1)$ and let $R = T(S)$ be the region that is the image of S under T.

(a) Check the box below the picture of R drawn against a dashed grid consisting of **unit squares**. **(2 points)**

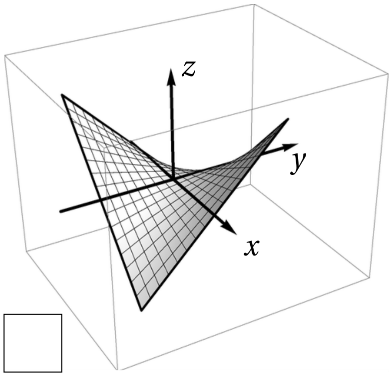


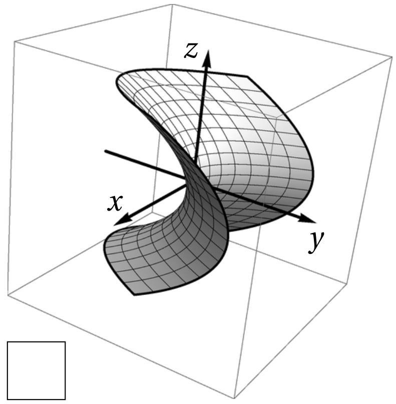
(b) Fill in the limits and integrand of the integral below so that it computes $\iint_R \cos(x+y)\, dA$ as an integral over the square S. **(4 points)**

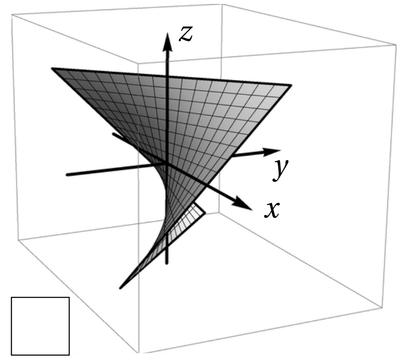
$$\iint_R \cos(x+y)\, dA = \int \qquad \int \qquad\qquad\qquad\qquad dv\, du$$

7. Let S be the surface parameterized by $\mathbf{r}(u,v) = \langle u,\ uv,\ v\rangle$ for $-1 \le u \le 1$ and $-1 \le v \le 1$.

(a) Mark the picture of S below. **(2 points)**







(b) Completely setup, but do not evaluate, the surface integral $\iint_S x^2\, dS$. **(5 points)**

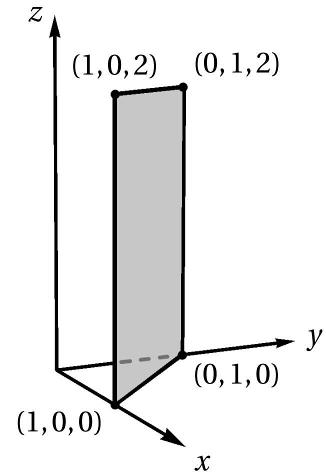
$$\iint_S x^2\, dS = \int_{-1}^{1}\int_{-1}^{1} \qquad \qquad du\, dv$$

(c) Circle the number closest to the area of S: 1 3 5 7 9 11 13 15 **(1 point)**

(d) Find the tangent plane to S at $(0,0,0)$. **(1 point)**

Equation: $\square\, x + \square\, y + \square\, z = \square$

8. Let R be the rectangle in the plane $x+y=1$ with vertices shown at right.

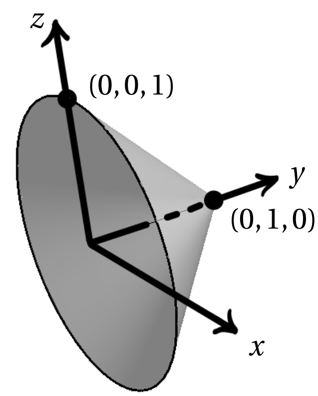


(a) Parameterize R by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u, v)-plane. **(3 points)**

$D = \{ \quad \}$	$\mathbf{r}(u,v) = \langle \quad , \quad , \quad \rangle$

(b) The integral $\iint_R z-1 \, dS$ is: | negative zero positive | **(1 point)**

9. Consider the cone C shown at right.



(a) Parameterize C by $\mathbf{r}\colon D \to \mathbb{R}^3$, being sure to specify the domain D of the parameterization in the (u, v)-plane. **(3 points)**

$D = \{ \quad \}$

$\mathbf{r}(u,v) = \langle \quad , \quad , \quad \rangle$

(b) The integral $\iint_C y \, dS$ is: | negative zero positive | **(1 point)**

Extra Credit: Consider the transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ which distorts the plane as shown below. Draw in $T(0,0)$ on the right-hand part of the picture and compute the Jacobian matrix of T at $(0,0)$, taking it as given that the entries of the matrix are integers and that the grid at left is made of unit squares. Be sure to explain your answer.

Note: If you need a makeshift ruler, you can tear off part of the upper right corner of this sheet.
(2 points)