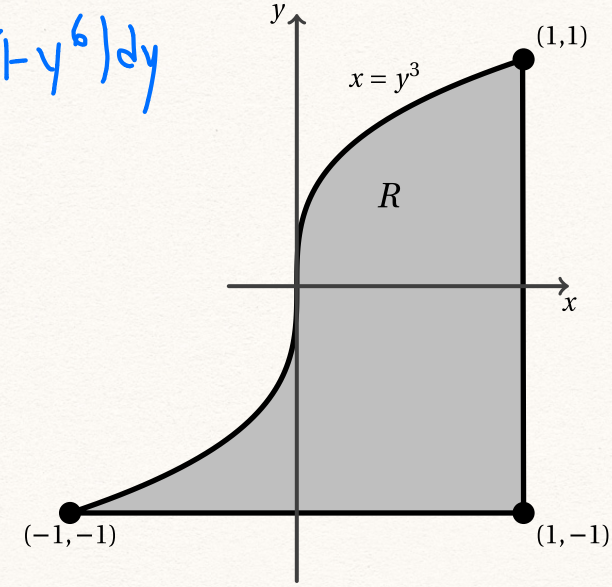
1. Let R denote the shaded region pictured below right. Compute $\iint_R 14x\,dA$. **(4 points)**

$y^3 \le x \le 1$

$-1 \le y \le 1$

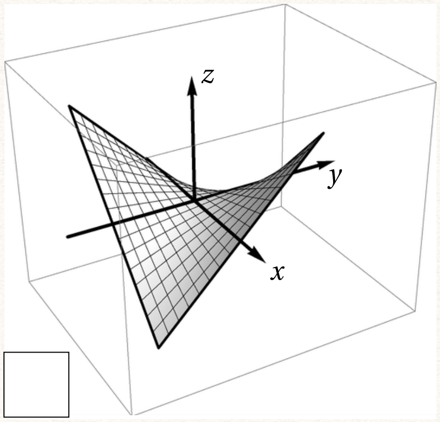
$\int_{-1}^{1}\int_{y^3}^{1} 14x\,dx\,dy = \int_{-1}^{1} 7(1-y^6)\,dy$

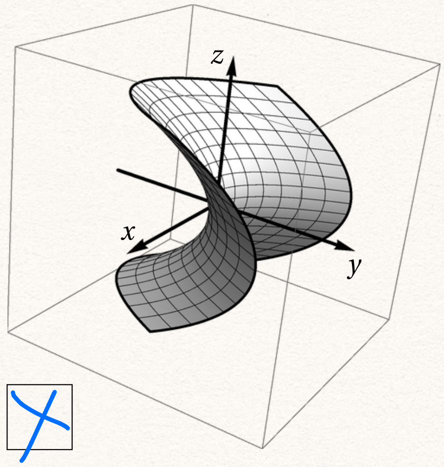
$= (7y - y^7)\Big|_{-1}^{1} = 14 - 2 = 12$

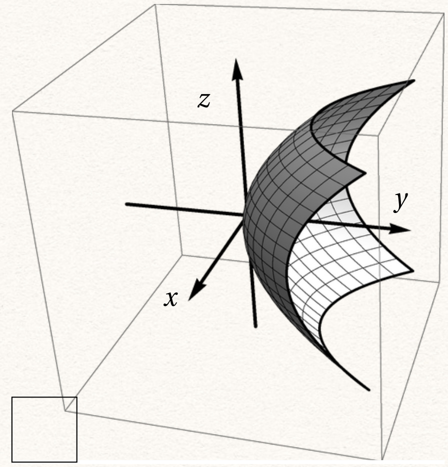


$\iint_R 14x\,dA = 12$

3. Let S be the surface parameterized by $\mathbf{r}(u,v) = \langle u, u^2 - v^2, v\rangle$ for $-1 \le u \le 1$ and $-1 \le v \le 1$. Mark the picture of S below. **(2 points)**







4. Consider the surface S parameterized by $\mathbf{r}(u,v) = (uv, v, u^2)$ with $-1 \le u \le 1$ and $0 \le v \le 1$.

(a) **(2 points)** Choose the integrand that correctly describes the integral over S of the function y, and circle your answer.

$\vec{r}_u = \langle v, 0, 2u\rangle$, $\vec{r}_v = \langle u, 1, 0\rangle$, $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v & 0 & 2u \\ u & 1 & 0 \end{vmatrix}$

$= \langle -2u, 2u^2, v\rangle$

$|\vec{r}_u \times \vec{r}_v| = (4u^2 + 4u^4 + v^2)^{1/2}$

$$\iint_S y\,dS = \int_0^1 \int_{-1}^1 \quad [\text{choice}] \quad dv\,du$$

$v\sqrt{4u^2+4u^4+v^2}$	$v\sqrt{u^2v^2+v^2+u^4}$
$u\sqrt{4u^2+4u^4+v^2}$	$v\sqrt{4u^2+4u^4+v}$

(b) **(2 points)** Circle the true statement:

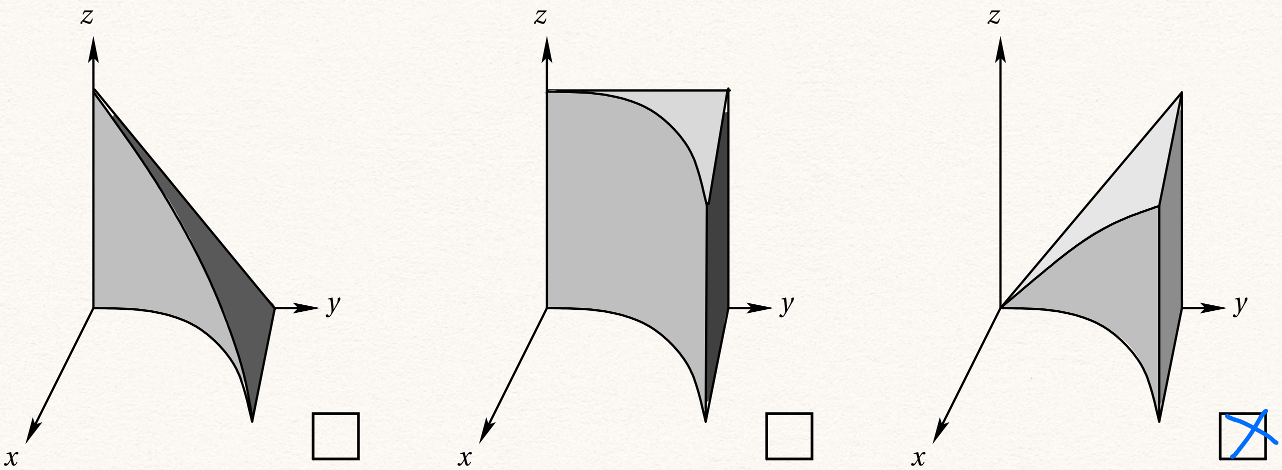
$\iint_S y\,dS < \iint_S y^2\,dS$	$\iint_S y\,dS = \iint_S y^2\,dS$	$\iint_S y\,dS > \iint_S y^2\,dS$

5. (a) **(3 points)** Calculate $\int_0^1 \int_0^{y^2} \int_0^y 2\,dz\,dx\,dy$.

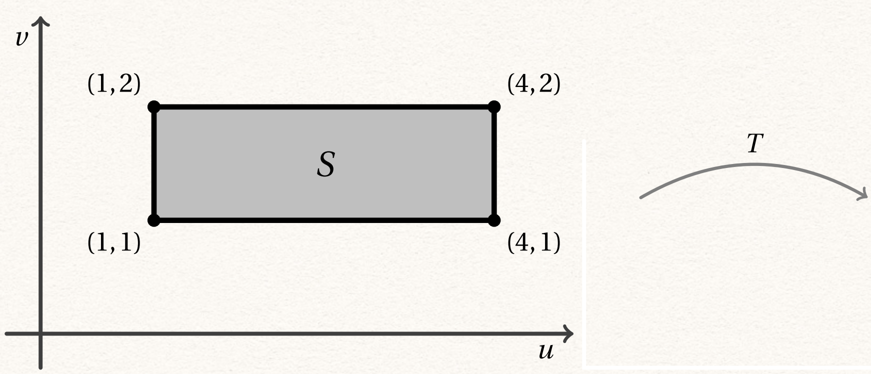
$$\int_0^1 \int_0^{y^2} 2y\,dx\,dy = \int_0^1 2y^3\,dy = \frac{1}{2}$$

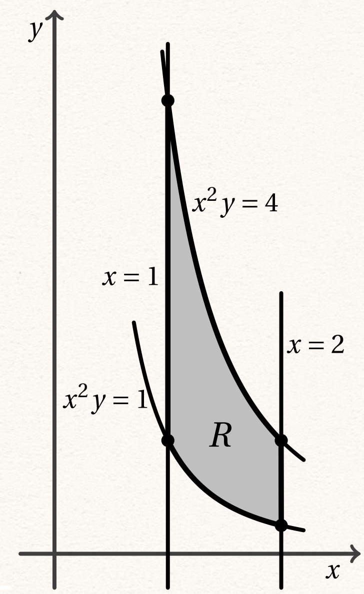
$$\boxed{\int_0^1 \int_{y^2}^1 \int_0^y 2\,dz\,dx\,dy = \ 1/2}$$

(b) **(2 points)** Decide which region is being integrated over in part (a), and check the corresponding box.



6. Let R be the region shown at right.





Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [1,4] \times [1,2]$ to R. Justify your answer. **(3 points)**

$u = x^2 y$ $\qquad$ $y = \frac{u}{v^2}$

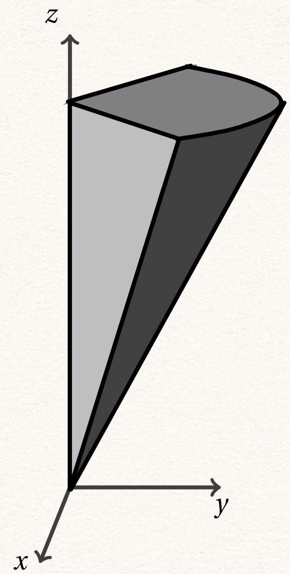
$v = x$

$$\boxed{T(u,v) = \left\langle \quad v \quad , \quad u/v^2 \quad \right\rangle}$$

7. Let R be the solid region in $\mathbb{R}^3$ shown at the right. Specifically, R is bounded by the cone $z^2 = 4x^2 + 4y^2$, the planes $x + y = 0$, $x - y = 0$, $z = 2$, and has $y, z \geq 0$. The volume of R is calculated in cylindrical coordinates by the integral

$$\int_A^B \int_C^D \int_E^F G\,dz\,dr\,d\theta.$$

Determine the quantities A, B, C, D, E, F, G and circle the correct answers.



(a) **(1 point)**

$A =$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$B =$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

(b) **(1 point)**

$C =$	0	1	2	3	4
$D =$	0	1	2	3	4

(c) **(1 point)** $E =$

0	$2r$	$4r^2$	2

(d) **(1 point)** $F =$

0	$2r$	$4r^2$	2

(e) **(1 point)** $G =$

0	1	r	r^2	$r\sin(\theta)$	$r^2\sin(\theta)$	$r^2\cos(\theta)$

8. Use spherical coordinates to SET UP, but not calculate, $\iiint_B z\,dV$, where B is the part of the unit ball in $\mathbb{R}^3$, $x^2 + y^2 + z^2 \leq 1$, for which $x \leq 0$. **(4 points)**

$$\iiint_B z\,dV = \int_{\pi/2}^{3\pi/2} \int_{0}^{\pi} \int_{0}^{1} \left(\rho^3 \sin\varphi \cos\varphi \right) d\rho\,d\phi\,d\theta$$

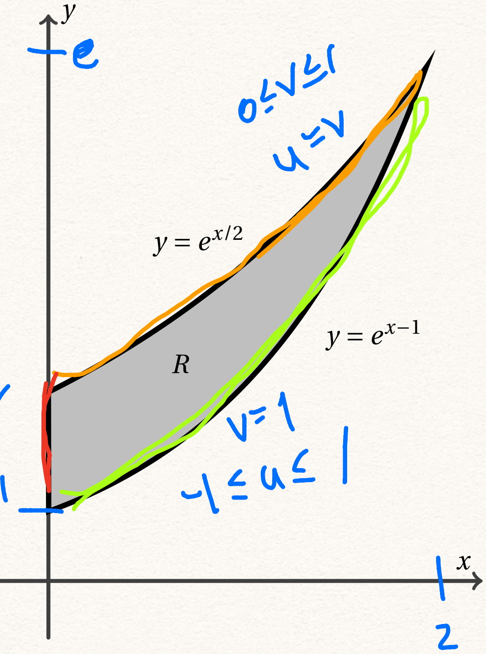
9. Let R be the region shown at right and consider the transformation

$$T(u,v) = \left(u+v, e^u\right).$$

Using the transformation to change coordinates, determine the quantities B, C, D, E, F in the expression

$$\iint_R \frac{x}{y}\, dA = \int_B^C \int_D^E \quad F \quad du\, dv$$

and **circle the correct answer**.



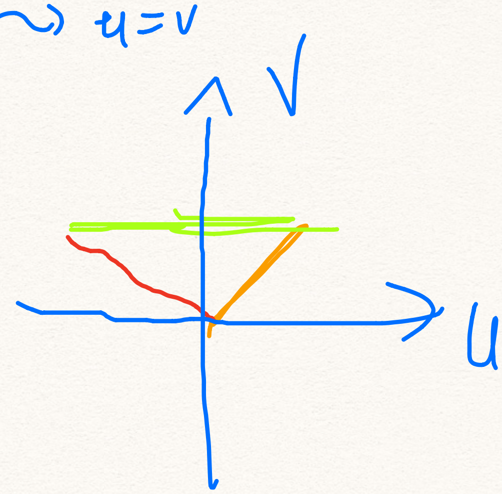
$$J(T) = \begin{pmatrix} 1 & 1 \\ e^u & 0 \end{pmatrix}$$

$0 = u + v$

$\det J(T) = -e^u$ $\qquad$ $|\det J(T)| = e^u$

$e^u = e^{u/2} e^{v/2} \rightsquigarrow e^{u/2} = e^{v/2} \rightsquigarrow u = v$

$e^u = e^{u+v-1} \rightsquigarrow v = 1.$



(a) **(1 point)** $B =$ | 0 | 1 | 2 | u | v | $-u$ | $-v$ |

(b) **(1 point)** $C =$ | 0 | 1 | 2 | u | v | $-u$ | $-v$ |

(c) **(1 point)** $D =$ | 0 | 1 | 2 | u | v | $-u$ | $-v$ |

(d) **(1 point)** $E =$ | 0 | 1 | 2 | u | v | $-u$ | $-v$ |

(e) **(2 points)** $F =$ | $(u+v)e^u$ | $(u+v)e^{-u}$ | $u+v$ | $-(u+v)e^u$ | $-(u+v)e^{-u}$ |

MORE PROBLEMS ON THE NEXT PAGE!

10. For each surface S in parts (a) and (b), give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D and call your parameters u and v.

(a) The part of the surface in $\mathbb{R}^3$ defined by $y = z^2 - x^2$ with $-1 \le x \le 1$ and $0 \le z \le 2$. **(3 points)**

$D = \{ -1 \le u \le 1$ and $0 \le v \le 2 \}$

$\mathbf{r}(u, v) = \langle u, v^2 - u^2, v \rangle$

(b) The portion of the sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$ where $z \lessgtr 0$. **(3 points)**

$D = \{ \frac{\pi}{2} \le u \le \pi$ and $0 \le v \le 2\pi \}$

$\mathbf{r}(u, v) = \langle \sin u \cos v, \sin u \sin v, \cos u \rangle$

(c) The portion of the surface in $\mathbb{R}^3$ defined by $z^2 - y^2 + 1 = 0$, $y \ge 0$ that lies inside the cylinder $x^2 + z^2 = 1$. **(3 points)**

$x = u\cos v$, $z = u \sin v$,

$y^2 = z^2 + 1$ or $y = \sqrt{z^2+1}$ (using $y \ge 0$)

$\rightsquigarrow y = \sqrt{u^2 \sin^2 v + 1}$

$D = \{ 0 \le u \le 1$ and $0 \le v \le 2\pi \}$

$\mathbf{r}(u, v) = \langle u\cos v, \sqrt{u^2\sin^2 v + 1}, u \sin v \rangle$